 Day 6 Review Polar and Parametric

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

D

1. A particle moves in the xy-plane such that its position for time $t \geq 0$ is given by $x(t) = 3t^2 - 19t$ and $y(t) = e^{2t-7}$. What is the slope of the tangent to the path of the particle when $t = 4$?
   (A) $-\frac{e}{28}$
   (B) $-\frac{28}{e}$
   (C) $\frac{e}{5}$
   (D) $\frac{2e}{5}$

$\frac{dy}{dt} = \frac{2e^{2t-7}}{6t-19}\Big|_{t=4} = \frac{2e}{5}$

A

2. The path of a particle in the xy-plane is given by the parametric equations $x(t) = \ln t$ and $y(t) = 5t^2 + 11$ for $t > 0$. An integral expression that represents the length of the path from $t = 2$ to $t = 6$ is
   (A) $\int_2^6 \sqrt{\frac{1}{t^2} + 100t^2}\, dt$
   (B) $\int_2^6 \sqrt{(\ln t)^2 + (5t^2 + 11)^2}\, dt$
   (C) $\int_2^6 \sqrt{1 + \frac{1}{t^2}}\, dt$
   (D) $\int_2^6 \sqrt{1 + 100t^2}\, dt$

$\int_2^6 \sqrt{\left(\frac{1}{t}\right)^2 + (10t)^2}\, dt$

A

3. The position vector of a particle moving in the xy-plane is $\left(t - \cos t,\ t^3 - 12t\right)$ for $t \in [0, 2\pi]$. For what value of y does the path of the particle have a horizontal tangent? $\frac{dy}{dx} = 0$
   (A) -16
   (B) $\frac{3\pi}{2}$
   (C) 2
   (D) 16

$3t^2 - 12 = 0$
$t = \pm 2$

$y = 2^3 - 12(2)$
$y = 8 - 24$

C

4. A plane curve has parametric equations $x(t) = t^2$ and $y(t) = t^4 + 3t^2$. An expression for the rate of change of the slope of the tangent to the path of the curve is
   (A) $2t^2 + 3$
   (B) $4t$
   (C) 2
   (D) $t^2 + 3$

slope $\frac{dy}{dx} = \frac{4t^3 + 6t}{2t} = 2t^2 + 3$

change in slope $\frac{d^2y}{dx^2} = \frac{d(2t^2+3)}{dx/dt} = \frac{4t}{2t}$

B

5. A particle moves in the xy-plane for $t > 0$ so that $x(t) = t^2 - 4t$ and $y(t) = \ln t$. At time $t = 1$, the particle is moving
   (A) to the right and up.
   (B) to the left and up.
   (C) to the left and down.
   (D) to the right and down.

$x' = 2t - 4$
$x'(1) = -2$
left

$y' = \frac{1}{x}$
$y'(1) = \frac{1}{1}$
up

D

6. The velocity vector of a particle moving in the xy-plane is $\left(\sqrt[3]{t},\ 6e^{2t-2}\right)$ for all real t. If the position of the particle at $t = 1$ is (0, 5), then the position vector of the particle is
   (A) $\left(t^{\frac{4}{3}} - 1,\ 3e^{2t-1} + 2\right)$
   (B) $\left(\frac{3}{4}t^{\frac{4}{3}},\ 3e^{2t-2}\right)$
   (C) $\left(\frac{3}{4}t^{\frac{4}{3}},\ 6e^{2t-2} - 1\right)$
   (D) $\left(\frac{3}{4}t^{\frac{4}{3}} - \frac{3}{4},\ 3e^{2t-2} + 2\right)$

$\int t^{1/3}\, dt$
$x = \frac{3t^{4/3}}{4} + C_x$
$0 = \frac{3}{4} + C_x$
$-\frac{3}{4} = C_x$

$\int 6e^{2t-2}$
$y = \frac{6}{2}e^{2t-2} + C_y$
$5 = 3e^0 + C_y$
$5 = 3 + C_y$
$2 = C_y$

C

7. A particle moving in the xy-plane has position vector $\left(e^{2t},\ \sqrt{t}\right)$ for $t \geq 0$. The acceleration vector of the particle is
   (A) $\left(4e^{2t},\ \frac{1}{4t^{\frac{3}{2}}}\right)$
   (B) $\left(e^{2t},\ -\frac{1}{4t^{\frac{3}{2}}}\right)$
   (C) $\left(4e^{2t},\ -\frac{1}{4t^{\frac{3}{2}}}\right)$
   (D) $\left(2e^{2t},\ \frac{1}{2\sqrt{t}}\right)$

$e^{2t},\ t^{1/2}$ pos
$2e^{2t},\ \frac{1}{2}t^{-1/2}$ vel
$4e^{2t},\ -\frac{1}{4}t^{-3/2}$ accel

A calculator may be used for the following questions.

8. A polar curve is given by $r = \frac{3}{2 - \cos\theta}$. The slope of the curve at $\theta = \frac{\pi}{2}$ is
   (A) 0
   (B) 0.5
   (C) 0.75
   (D) -0.75

$\frac{dy}{dx} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$

have the calculator do it!

No time to do it by hand

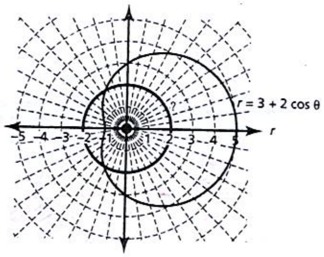
B

9. The position vector of a particle moving in the xy-plane is $(t^2,\ \sin t)$. What is the distance traveled by the particle from $t = 0$ to $t = \pi$?
(A) 9.870
(B) 10.354
(C) 12.335
(D) 42.912

$\int_0^{\pi} \sqrt{(2t)^2 + (\cos t)^2}\, dt$

D

10. The area inside the polar curve $r = 3 + 2\cos\theta$ is
(A) 9.425
(B) 18.850
(C) 28.274
(D) 34.558



$A = \frac{1}{2}\int_0^{2\pi} (3 + 2\cos\theta)^2\, d\theta$

D

11. A particle moves in the xy-plane so that its acceleration vector for time $t > 0$ is $\left(6t^2, \frac{20}{t}\right)$. If the velocity vector at $t = 1$ is $(5,0)$, then how fast is the particle moving when $t = 3$?
(A) 36.069
(B) 54.410
(C) 58.299
(D) 61.088

speed

$\int 6t^2\,dt \quad \int 20t^{-1}\,dt$

$x = \frac{6t^3}{3} + C_x \quad y = 20\ln t + C_y$

$5 = 2 + C_x \quad 0 = 20\cdot 0 + C_y$

$3 = C_x \quad 0 = C_y$

A calculator may not be used on the following questions.

C

12. A particle moves in the xy-plane so that its velocity for time $t \geq 0$ is given by the parametric equations $x'(t) = e^{2t}$ and $y'(t) = \sqrt{3t+1}$. An expression for the distance traveled by the particle on the interval $t \in [1,5]$ is
(A) $\int_1^5 \left(e^{4t} + 3t + 1\right) dt$
(B) $\int_1^5 \left(e^{2t} + \sqrt{3t+1}\right)^2 dt$
(C) $\int_1^5 \sqrt{e^{4t} + 3t + 1}\, dt$
(D) $\int_1^5 \left(\frac{1}{2}e^{2t} + \frac{2}{9}(3t+1)^{\frac{3}{2}}\right) dt$

$\int_1^5 \sqrt{(e^{2t})^2 + \left(\sqrt{3t+1}\right)^2}\, dt$

A

13. The area enclosed inside the polar curve $r^2 = 10\cos(2\theta)$ is
(A) 10
(B) 5π
(C) 20
(D) 10π

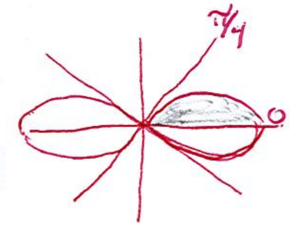
No calculator

$4\left[\int_0^{\pi/4} \frac{1}{2}\cdot 10\cos(2\theta)\, d\theta\right]$

$4\cdot 5\int \cos(2\theta)\, d\theta = \frac{20}{2}\sin 2\theta \Big]_0^{\pi/4}$

$10\sin\frac{\pi}{2} - 10\sin 0$

$10 - 0$



(11) $v(3) = \langle 2(3)^3 + 3,\ 20\ln 3\rangle$

speed $= \sqrt{57^2 + (20\ln 3)^2}$

A

~~14. A~~ particle moves in the xy-plane so that its velocity vector for time $0 \leq t \leq 10$ is $\left(\sqrt{100 - 10t}, 2t\right)$. Which one of the following statements is true about the particle when $t = 1$?
(A) The particle is slowing down.
~~(B)~~ The particle is speeding up.
(C) The particle is at rest.
(D) The speed of the particle is $\sqrt{90} + 2$.

$v(1) \langle \sqrt{90}, 2\rangle$

Speed $= \sqrt{(\sqrt{100 - 10t})^2 + (2t)^2}$

speed $= \sqrt{100 - 10t + 4t^2}$

$\frac{d}{dt}(\text{speed})$

$= \frac{1}{2}(100 - 10t + 4t^2)^{-1/2} \cdot (-10 + 8t)$

$= \frac{8t - 10}{2\sqrt{100 - 10t + 4t^2}}$

$t = 1$

$= \frac{-2}{2\sqrt{94}}$

speed dec

B

15. A particle moves in the xy-plane for values of time t in the interval $1.25 \leq t \leq 3.75$ according to the parametric equations $x(t) = \cos(\pi t)$ and $y(t) = t^3 - 24t^2 + 45t$. For what value(s) of t in this interval is the line tangent to the path of the particle vertical?
(A) 3 only
(B) 2 and 3 only
(C) 1.5, 2.5, and 3.5 only
(D) 1.5, 2.5, 3, and 3.5 only

FREE-RESPONSE QUESTION

A calculator may be used on this question.

1. A particle moving in the xy-plane has acceleration vector $\left(\sqrt{t}, \frac{1}{1+t^2}\right)$ for all $t \geq 0$. The particle is at rest at time $t = 0$.
(a) Give the velocity vector of the particle at time $t = 0$.
(b) Give the velocity vector of the particle at time $t = 3$.
(c) How fast is the particle moving at time $t = 3$?
(d) What is the total distance traveled by the particle in the time interval $0 \leq t \leq 3$?

(15) $\frac{dy}{dx}$ undefined

$\frac{dx}{dt} = -\pi\sin(\pi t) = 0$

$\sin(\pi t) = 0$

$t = 0, 1, 2, 3, \ldots$

$1.25 \leq t \leq 3.75$

14. ANSWER: (A) The speed of the particle is $\sqrt{100-10t+4t^2}$.

$$\frac{d}{dt}[speed]\bigg|_{t=1} = \frac{-10+8t}{2\sqrt{100-10t+4t^2}}\bigg|_{t=1} = \frac{-2}{2\sqrt{94}} < 0.$$

Since the derivative of speed is negative, the speed is decreasing. In other words, the particle is slowing down. (*Calculus for AP* 1st ed. pages 653–655; EU 2.3; LO 2.3C; EK 2.3C4; MPAC 2)

15. ANSWER: (B) $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2-24t+45}{-\pi\sin(\pi t)}.$$ The path has a vertical tangent when $\frac{dx}{dt} = -\pi\sin(\pi t) = 0 \Rightarrow t = 0, 1, 2, 3, \ldots$

For $1.25 \le t \le 3.75$ the solutions are $t = 2$ and 3. (*Calculus for AP* 1st ed. pages 653–655; EU 2.1,2.3; LO 2.1C,2.3B; EK 2.1C7,2.3B1; MPAC 2,3)

FREE-RESPONSE QUESTION

	Solution	Possible points
(a)	The velocity vector at time $t = 0$ is (0, 0), because when the particle is at rest, both components of the velocity vector must be zero.	1: answer with reason
(b)	$\int\sqrt{t}\ dt = \frac{2}{3}t^{\frac{3}{2}} + C_1 \Rightarrow \frac{2}{3}(0)^{\frac{3}{2}} + C_1 = 0 \Rightarrow C_1 = 0$ $\int\frac{1}{1+t^2}\ dt = \tan^{-1}t + C_2 \Rightarrow \tan^{-1}(0) + C_2 = 0, C_2 = 0$ The velocity vector at $t = 3$ is $\left(\frac{2}{3}(3)^{\frac{3}{2}},\ \tan^{-1}3\right) \approx (3.464,\ 1.249)$	4: $\begin{cases}\text{1: antiderivative of } \sqrt{t} \\ \text{1: antiderivative of } \frac{1}{1+t^2} \\ \text{1: both constants of integration} \\ \quad \langle -1 \rangle \text{ first error} \\ \text{1: answer}\end{cases}$
(c)	$\sqrt{\left(\frac{2}{3}3^{\frac{3}{2}}\right)^2 + \left(\tan^{-1}3\right)^2} = 3.682$	2: $\begin{cases}\text{1: expression for speed} \\ \text{1: answer}\end{cases}$
(d)	$\int_0^3\sqrt{\left(\frac{2}{3}t^{\frac{3}{2}}\right)^2 + \left(\tan^{-1}t\right)^2}\ dt = 5.006$	2: $\begin{cases}\text{1: integrand with limits} \\ \text{1: answer}\end{cases}$

(a), (b), (c), (d) (*Calculus for AP* 1st ed. pages 653–656; EU 2.3,3.4; LO 3.4C,3.4D,2.3; EK 3.4C2,2.3C4,3.4D3; MPAC 2,3)