 Day 6 Review Polar and Parametric

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

1. A particle moves in the xy-plane such that its position for time $t \geq 0$ is given by $x(t) = 3t^2 - 19t$ and $y(t) = e^{2t-7}$. What is the slope of the tangent to the path of the particle when $t = 4$?

   (A) $-\dfrac{e}{28}$

   (B) $-\dfrac{28}{e}$

   (C) $\dfrac{e}{5}$

   (D) $\dfrac{2e}{5}$

2. The path of a particle in the xy-plane is given by the parametric equations $x(t) = \ln t$ and $y(t) = 5t^2 + 11$ for $t > 0$. An integral expression that represents the length of the path from $t = 2$ to $t = 6$ is

   (A) $\displaystyle\int_2^6 \sqrt{\frac{1}{t^2} + 100t^2}\, dt$

   (B) $\displaystyle\int_2^6 \sqrt{(\ln t)^2 + (5t^2 + 11)^2}\, dt$

   (C) $\displaystyle\int_2^6 \sqrt{1 + \frac{1}{t^2}}\, dt$

   (D) $\displaystyle\int_2^6 \sqrt{1 + 100t^2}\, dt$

3. The position vector of a particle moving in the xy-plane is $\left(t - \cos t,\ t^3 - 12t\right)$ for $t \in [0, 2\pi]$. For what value of y does the path of the particle have a horizontal tangent?

   (A) -16

   (B) $\dfrac{3\pi}{2}$

   (C) 2

   (D) 16

4. A plane curve has parametric equations $x(t) = t^2$ and $y(t) = t^4 + 3t^2$. An expression for the rate of change of the slope of the tangent to the path of the curve is

   (A) $2t^2 + 3$

   (B) $4t$

   (C) 2

   (D) $t^2 + 3$

5. A particle moves in the xy-plane for $t > 0$ so that $x(t) = t^2 - 4t$ and $y(t) = \ln t$. At time $t = 1$, the particle is moving
   (A) to the right and up.
   (B) to the left and up.
   (C) to the left and down.
   (D) to the right and down.

6. The velocity vector of a particle moving in the xy-plane is $\left(\sqrt[3]{t},\ 6e^{2t-2}\right)$ for all real t. If the position of the particle at $t = 1$ is (0, 5), then the position vector of the particle is
   (A) $\left(t^{\frac{4}{3}} - 1,\ 3e^{2t-1} + 2\right)$
   (B) $\left(\frac{3}{4}t^{\frac{4}{3}},\ 3e^{2t-2}\right)$
   (C) $\left(\frac{3}{4}t^{\frac{4}{3}},\ 6e^{2t-2} - 1\right)$
   (D) $\left(\frac{3}{4}t^{\frac{4}{3}} - \frac{3}{4},\ 3e^{2t-2} + 2\right)$

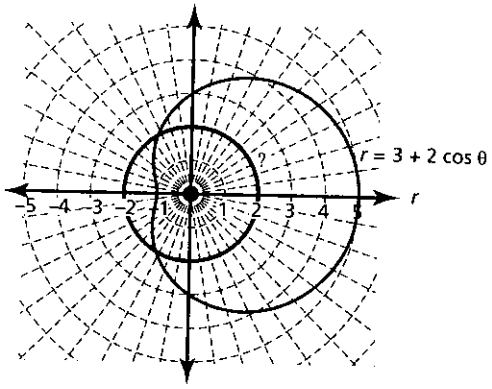
7. A particle moving in the xy-plane has position vector $\left(e^{2t},\ \sqrt{t}\right)$ for $t \geq 0$. The acceleration vector of the particle is
   (A) $\left(4e^{2t},\ \frac{1}{4t^{\frac{3}{2}}}\right)$
   (B) $\left(e^{2t},\ -\frac{1}{4t^{\frac{3}{2}}}\right)$
   (C) $\left(4e^{2t},\ -\frac{1}{4t^{\frac{3}{2}}}\right)$
   (D) $\left(2e^{2t},\ \frac{1}{2\sqrt{t}}\right)$

A calculator may be used for the following questions.

8. A polar curve is given by $r = \frac{3}{2 - \cos\theta}$. The slope of the curve at $\theta = \frac{\pi}{2}$ is
   (A) 0
   (B) 0.5
   (C) 0.75
   (D) −0.75

9. The position vector of a particle moving in the xy-plane is $\left(t^2,\ \sin\ t\right)$. What is the distance traveled by the particle from $t = 0$ to $t = \pi$?
(A) 9.870
(B) 10.354
(C) 12.335
(D) 42.912

10. The area inside the polar curve $r = 3 + 2\cos\theta$ is
(A) 9.425
(B) 18.850
(C) 28.274
(D) 34.558



11. A particle moves in the xy-plane so that its acceleration vector for time $t > 0$ is $\left(6t^2, \frac{20}{t}\right)$. If the velocity vector at $t = 1$ is $(5,0)$, then how fast is the particle moving when $t = 3$?
(A) 36.069
(B) 54.410
(C) 58.299
(D) 61.088

A calculator may not be used on the following questions.

12. A particle moves in the xy-plane so that its velocity for time $t \geq 0$ is given by the parametric equations $x'(t) = e^{2t}$ and $y'(t) = \sqrt{3t+1}$. An expression for the distance traveled by the particle on the interval $t \in [1,5]$ is
(A) $\int_1^5 \left(e^{4t} + 3t + 1\right) dt$
(B) $\int_1^5 \left(e^{2t} + \sqrt{3t+1}\right)^2 dt$
(C) $\int_1^5 \sqrt{e^{4t} + 3t + 1}\, dt$
(D) $\int_1^5 \left(\frac{1}{2}e^{2t} + \frac{2}{9}(3t+1)^{\frac{3}{2}}\right) dt$

13. The area enclosed inside the polar curve $r^2 = 10\cos(2\theta)$ is
(A) 10
(B) 5π
(C) 20
(D) 10π

14. A particle moves in the xy-plane so that its velocity vector for time $0 \le t \le 10$ is $\left(\sqrt{100-10t}, 2t\right)$. Which one of the following statements is true about the particle when $t = 1$?
(A) The particle is slowing down.
(B) The particle is speeding up.
(C) The particle is at rest.
(D) The speed of the particle is $\sqrt{90}+2$.

15. A particle moves in the xy-plane for values of time t in the interval $1.25 \le t \le 3.75$ according to the parametric equations $x(t) = \cos(\pi t)$ *and* $y(t) = t^3 - 24t^2 + 45t$. For what value(s) of t in this interval is the line tangent to the path of the particle vertical?
(A) 3 only
(B) 2 and 3 only
(C) 1.5, 2.5, and 3.5 only
(D) 1.5, 2.5, 3, and 3.5 only

FREE-RESPONSE QUESTION

A calculator may be used on this question.

1. A particle moving in the xy-plane has acceleration vector $\left(\sqrt{t}, \frac{1}{1+t^2}\right)$ for all $t \ge 0$. The particle is at rest at time $t = 0$.
(a) Give the velocity vector of the particle at time $t = 0$.
(b) Give the velocity vector of the particle at time $t = 3$.
(c) How fast is the particle moving at time $t = 3$?
(d) What is the total distance traveled by the particle in the time interval $0 \le t \le 3$?