

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used for the following questions.

1. Which of the following series is absolutely convergent?

(A) $\sum_{k=0}^{\infty} (-1)^k \frac{k+3}{k+\sqrt{k}}$

(B) $\sum_{k=0}^{\infty} (-1)^k \frac{3}{\sqrt{k}}$

(C) $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{\sqrt{k}}{k+3}$

(D) $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{3}{k\sqrt{k}}$

2. The power series $\sum_{k=1}^{\infty} \frac{x^k}{k}$ converges for which values of x ?

(A) $-1 < x < 1$

(B) $-1 \leq x < 1$

(C) $-1 \leq x \leq 1$

(D) x is any real number.

3. Which of the following series is the power series expansion for $f(x) = x(\cos x - 1)$?

(A) $x - \frac{x^3}{2} + \frac{x^5}{24} - \dots$

(B) $-x^3 + x^5 - x^7 + \dots$

(C) $\frac{x^3}{2} - \frac{x^5}{24} + \frac{x^7}{720} - \dots$

(D) $-\frac{x^3}{2} + \frac{x^5}{24} - \frac{x^7}{720} + \dots$

4. What are all values of a for which the series $\sum_{k=0}^{\infty} \left(\frac{5}{9-a}\right)^k$ converges?

(A) $a < 4$

(B) $4 < a < 14$

(C) $a < 9$

(D) $a < 4$ or $a > 14$

5. The Maclaurin series for $f(x) = \frac{1}{1+x^2}$ is $\sum_{k=0}^{\infty} (-1)^k x^{2k}$. What is the Maclaurin series for $g(x) = \tan^{-1} x$?

(A) $C + \sum_{k=0}^{\infty} (-1)^k (2k) x^{2k-1}$

(B) $C + \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$

(C) $C + \sum_{k=0}^{\infty} (-1)^{k+1} \frac{x^{2k+1}}{2k}$

(D) $C + \sum_{k=0}^{\infty} (-1)^{k-1} \frac{x^{2k-1}}{2k}$

6. The Maclaurin series $\sum_{k=0}^{\infty} (-9)^k \frac{x^{4k}}{(2k)!}$ represents which function below?

(A) $\cos(3x^2)$

(B) $\sin(3x^2)$

(C) $\cos(9x^4)$

(D) e^{-3x^2}

7. If the first five terms of the Taylor expansion for $f(x)$ about $x = 0$ are

$$3 - 7x + \frac{5}{2}x^2 + \frac{3}{4}x^3 - 6x^4, \text{ then } f'''(0) =$$

(A) $\frac{1}{8}$

(B) $\frac{3}{4}$

(C) $\frac{9}{2}$

(D) 6

8. Which of the following series diverge?

I. $\sum_{k=0}^{\infty} \frac{k^{\frac{3}{2}} + 1}{5k^2 + 7}$

II. $\sum_{k=2}^{\infty} (-1)^k \frac{1}{\ln(k)}$

III. $\sum_{k=0}^{\infty} (-1)^k \left(\frac{4}{3}\right)^k$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

A calculator may be used for the following questions.

9. The sixth degree term of the Taylor series expansion for $f(x) = e^{-\frac{1}{2}x^2}$ about $x = 0$ has coefficient
- (A) $-\frac{1}{48}$
 (B) $-\frac{1}{6}$
 (C) $\frac{1}{720}$
 (D) $-\frac{1}{4608}$
10. Let $T_n(x)$ represent the Taylor Polynomial of degree n about $x = 0$ for $f(x) = e^{-x}$. If $T_n(2)$ is used to approximate the value of $f(2) = e^{-2}$, then which of the following expressions is NOT less than $\frac{2^7}{7!}$?
- (A) $f(2) - T_6(2)$
 (B) $T_6(2) - f(2)$
 (C) $f(2) - T_5(2)$
 (D) $T_5(2) - f(2)$
- *11. For function $f(x)$, $f(0) = 3$, $f'(0) = 2$, $f''(0) = 5$, and $f'''(0) = 4$. Using the Taylor series expansion for $f(x)$ about $x = 0$, the second degree estimate for $f'(0.1)$ is
- (A) 2.500
 (B) 2.520
 (C) 3.120
 (D) 3.225

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12. The Maclaurin series $\sum_{k=0}^{\infty} (-5)^k \frac{x^{3k+2}}{k!}$ represents which expression below?
- (A) $x^2 e^{-5x^3}$
 (B) $-5e^{3x+2}$
 (C) $x^2 \sin(-5x^3)$
 (D) $-5\cos(x^3) + 2$

13. If $f(x) = \sin(x^2)$, the first three terms of the Taylor series expansion about $x = 0$ for $f'(x)$ are

- (A) $2x - x^3 + \frac{1}{2}x^5$
 (B) $1 - \frac{1}{2}x^4 + \frac{1}{16}x^8$
 (C) $2x - x^5 + \frac{1}{12}x^9$
 (D) $x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10}$

14. Which of the following series is conditionally convergent?

- (A) $\sum_{k=1}^{\infty} (-1)^k \frac{k^2 + 1}{k + 4}$
 (B) $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{2}{k + 1}$
 (C) $\sum_{k=1}^{\infty} (-1)^{k+1} \ln(k + 1)$
 (D) $\sum_{k=1}^{\infty} (-1)^k \frac{k + 3}{k^3 + 7}$

15. The radius of convergence of the power series $\sum_{k=0}^{\infty} \frac{x^{2k}}{k \cdot 4^k}$ is

- (A) 0
 (B) 1
 (C) 2
 (D) 4

FREE-RESPONSE QUESTION

A calculator may be used for this question.

Let $f(x)$ be a function that is differentiable for all x . The Taylor expansion for $f(x)$ about $x = 0$ is given by $T(x) = \sum_{k=0}^{\infty} (-1)^k \frac{(k+1)x^{3k}}{k!}$. The

first four nonzero terms of $T(x)$ are given by $T_4(x) = 1 - 2x^3 + \frac{3x^6}{2!} - \frac{4x^9}{3!}$.

- (a) Show that $T(x)$ converges for all x .
 (b) Let $W(x)$ be the Taylor expansion for $x^2 f'(x)$ about $x = 0$. Find the general term for $W(x)$ and find $W_3(x)$.
 (c) $f(0.5) \approx T_4(0.5)$ and $f(1) \approx T_4(1)$. Find the values of $T_4(0.5)$ and $T_4(1)$.
 (d) Which value is smaller, $|f(0.5) - T_4(0.5)|$ or $|f(1) - T_4(1)|$? Give a reason for your answer.