Review #1

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

1. Evaluate the limit, if it exists: $\lim_{x\to 2}\dfrac{x^2+x-6}{2-x}$.

   (A) 5
   (B) 3
   (C) –5
   (D) The limit does not exist.

2. Evaluate the limit, if it exists: $\lim_{x\to 9}\dfrac{\sqrt{x-5}-2}{x-9}$.

   (A) $\dfrac{1}{4}$
   (B) $-\dfrac{1}{4}$
   (C) 0
   (D) The limit does not exist.

3. Evaluate the limit, if it exists: $\lim_{x\to 2}\dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$.

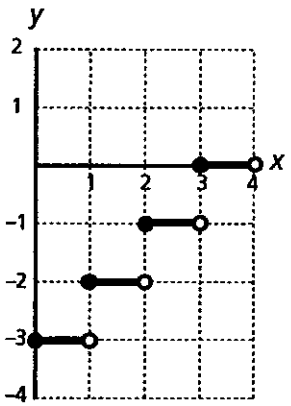
   (A) $\dfrac{1}{4}$
   (B) $-\dfrac{1}{4}$
   (C) 1
   (D) The limit does not exist.

4. Evaluate the limit, if it exists: $\lim_{x\to 1}\dfrac{\tan^{-1}x}{\sin^{-1}x+1}$.

   (A) $\dfrac{1}{4}$
   (B) $\dfrac{1}{2}$
   (C) $\dfrac{\pi}{2}$
   (D) $\dfrac{\pi}{2\pi+4}$

5. Estimate the limit, if it exists: $\lim_{x\to 3} f(x)$ where $f(x)$ is represented by the given graph:

   (A) 0
   (B) –1
   (C) 1
   (D) The limit does not exist.



6. Given the function

$$f(x)=\begin{cases}\sin 2x, & x \le \pi \\ 2x+k, & x > \pi\end{cases}$$

what value of k will make this piecewise function continuous?

(A) -2π
(B) $-\pi$
(C) π
(D) 2π

7. Find the limit, if it exists: $\lim_{x\to 0} x\left(e^x+\frac{1}{x}\right)$.

(A) 0
(B) 1
(C) 2
(D) The limit does not exist.

8. Identify the vertical asymptotes for $f(x)=\frac{x^2+3x-4}{x^2+x-2}$.

(A) $x=-2, x=1$
(B) $x=-2$
(C) $x=1$
(D) $x=-2, x=1$

9. If $p(x)$ is a continuous function on the closed interval [1, 3], with $p(1) \le K \le p(3)$ and c is in the closed interval [1, 3], then which of the following statements must be true?

(A) $p(c)=\frac{p(3)+p(1)}{2}$
(B) $p(c)=\frac{p(3)-p(1)}{2}$
(C) There is at least one value c such that $p(c)=K$.
(D) There is only one value c such that $p(c)=K$.

10. How many vertical asymptotes exist for the function $f(x)=\frac{1}{2\sin^2 x-\sin x-1}$ in the open interval $0<x<2\pi$?

(A) 0
(B) 1
(C) 2
(D) 3

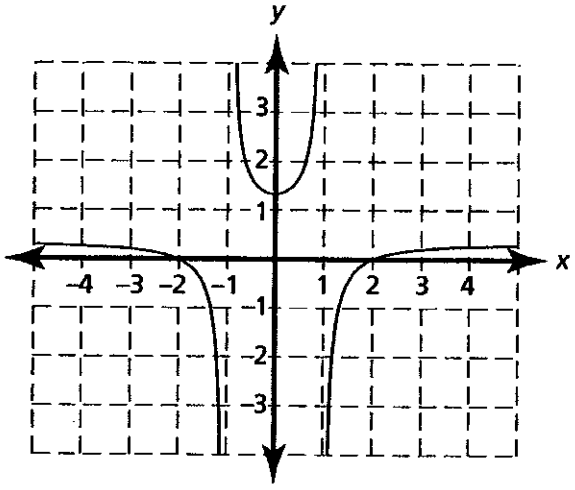
11. Selected values for continuous functions $f(x)$ and $g(x)$ are given in the table above. $\lim_{x\to 3} \frac{f(g(x))}{g(f(x))} =$

x	1	2	3	4
$f(x)$	4	2	3	1
$g(x)$	2	3	1	4

(A) $\frac{1}{4}$
(B) $\frac{1}{3}$
(C) 3
(D) 4

12. $\lim_{x\to\infty} \frac{\sin x}{e^x + \cos x} =$
(A) -1
(B) 0
(C) $\frac{1}{e}$
(D) 1

13. For what value of k is the function $f(x) = \begin{cases} \frac{2x^2+5x-3}{x^2-9}, & x \neq -3 \\ k, & x = -3 \end{cases}$ continuous at $x = -3$?
(A) $-\frac{7}{6}$
(B) 0
(C) $\frac{5}{6}$
(D) $\frac{7}{6}$



14. The function $g(x)$ is shown in the graph above and is of the form $g(x) = \frac{x^2+a}{bx^2-3}$. Which of the following could be the values of the constants a and b?
(A) $a = -2,\ b = -1$
(B) $a = -2,\ b = -3$
(C) $a = -4,\ b = 3$
(D) $a = -4,\ b = -3$

15. Which of the following statements is true?
(A) $\lim_{x\to 3} \log_3 x = 2$
(B) $\lim_{x\to 0^+} \log_3 x$ does not exist.
(C) $\lim_{x\to -\infty} e^x$ does not exist.
(D) $\lim_{x\to 1} e^{x-1} = 0$

MULTIPLE-CHOICE QUESTIONS

Calculators may not be used for the following questions.

1. What does the limit statement $\lim_{x\to 1}\frac{\ln(x+1)-\ln 2}{x-1}$ represent?

   (A) 0

   (B) $\frac{d}{dx}[\ln(x+1)]$

   (C) $f'(1)$, if $f(x)=\ln(x+1)$

   (D) 1

2. Find the derivative of the function $y=\frac{4}{x^3}$.

   (A) $-\frac{12}{x^2}$

   (B) $\frac{12}{x^2}$

   (C) $\frac{12}{x^4}$

   (D) $-\frac{12}{x^4}$

3. Find $\frac{dy}{dx}$ if $3xy=4x+y^2$.

   (A) $\frac{4-3y}{2y-3x}$

   (B) $\frac{3x-4}{2x}$

   (C) $\frac{3y-x}{2}$

   (D) $\frac{3y-4}{2y-3x}$

4. Find $\frac{dy}{dx}$ for $e^{x+y}=y$.

   (A) $\frac{e^{x+y}}{(1-e^{x+y})}$

   (B) $\frac{e^{x+y}}{(1+e^{x+y})}$

   (C) $\frac{e^{x+y}}{(e^{x+y}-1)}$

   (D) e^{x+y}

5. If the nth derivative of y is denoted as $y^{(n)}$ and $y=-\sin x$, then $y^{(7)}$ is the same as

   (A) y

   (B) $\frac{dy}{dx}$

   (C) $\frac{d^2y}{dx^2}$

   (D) $\frac{d^3y}{dx^3}$

6. Find the second derivative of $f(x)$ if $f(x) = (2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $24(2x+3)^2$

(D) $48(2x+3)^2$

Calculators may be used for the following questions.

7. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.

(A) $8\sin(3x)$

(B) $8\sin(3x)\cos(3x)$

(C) $12\sin(3x)\cos(3x)$

(D) $24\sin(3x)\cos(3x)$

9. If $\ln y = (\ln x)^2 + 2$, find $\frac{dy}{dx}$ in terms of x and y.

(A) $y\left[2\ln(x)+\frac{1}{x}\right]$

(B) $y\left[\left(\frac{2}{x}\right)\ln(x)\right]$

(C) $\left(\frac{2}{x}\right)\ln(x)$

(D) $y\left[\frac{2(\ln x)}{x}+2\right]$

10. If $f(2) = -3$, $f'(2) = \frac{3}{4}$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

(A) $y-2 = \frac{-3}{4}(x+3)$

(B) $y-2 = \frac{-4}{3}(x+3)$

(C) $y+2 = \frac{4}{3}(x-3)$

(D) $y-2 = \frac{4}{3}(x+3)$

11. For what positive value of x does the tangent line to the curve $y = \ln(1-x)$ intersect the y-axis at the point (0, 2)?

(A) 0.382

(B) 0.547

(C) 0.667

(D) 0.778

A calculator may not be used for the following questions.

12. For what values of a and c is the piecewise function $f(x) = \begin{cases} ax^2+\sin x, & x \le \pi \\ 2x-c, & x > \pi \end{cases}$ differentiable?

(A) $a = \frac{3\pi}{2}$ and $c = \frac{\pi}{2}$

(B) $a = \frac{3}{2\pi}$ and $c = \frac{7\pi}{2}$

(C) $a = \frac{3}{2\pi}$ and $c = -\frac{\pi}{2}$

(D) $a = \frac{3}{2\pi}$ and $c = \frac{\pi}{2}$

13. If $y = \tan^{-1}(x^2 + 3x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+(x^2+3x)^2}$

(B) $\frac{1}{x^2+3x+1}$

(C) $\frac{2x+3}{1+(x^2+3x)^2}$

(D) $\frac{2x+3}{(x^2+3x)^2}$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	1	–2	4
2	5	3	1	–4
3	2	1	–2	1
4	4	–3	2	–1

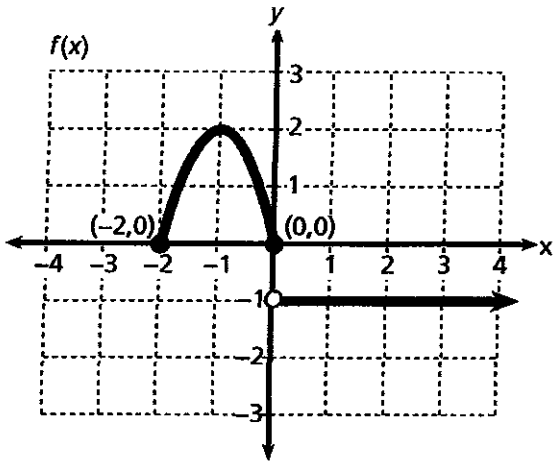
14. Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are given in the table above. If $p(x) = x \cdot f(x) - g(3x-2)$, then $p'(2) =$

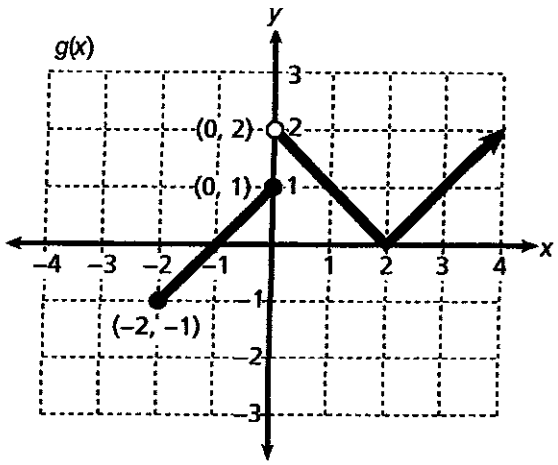
(A) 11
(B) 10
(C) 8
(D) 6

FREE-RESPONSE QUESTION

A calculator may not be used for this question.

1. Use the graphs of $f(x)$ and $g(x)$ given below to answer the following questions:





(a) Is $f[g(x)]$ continuous at $x = 0$? Explain why or why not.
(b) Is $g[f(x)]$ continuous at $x = 0$? Explain why or why not.
(c) What is $\lim_{x \to \infty} f[g(x)]$? Explain your reasoning.
(d) If $h(x) = \begin{cases} f(x)+g(x), & -2 \le x \le 0 \\ k+g(x)f(x), & x > 0 \end{cases}$, what is k so that $h(x)$ is continuous at $x = 0$?