1

① $V = \frac{4}{3}\pi r^3$ $\frac{dr}{dt} = \frac{1}{\pi}$ $r = 1$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\cancel{\pi} \cdot \cancel{3} \cdot \frac{1}{\cancel{\pi}} = \boxed{4\ in^3/sec}$ (C)

② $A = s^2$ $\frac{ds}{dt} = 2\ ft/min$ $s = 7$

$\frac{dA}{dt} = 2s\frac{ds}{dt}$

$\frac{dA}{dt} = 2(7)(2) = \boxed{28\ ft^2/sec}$ (A)

$y = 3(x^2+4)^{-1}$

③ $\frac{dy}{dt} = -3(x^2+4)^{-2} \cdot 2x\frac{dx}{dt}$

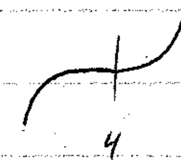
$6 = -3(4+4)^{-2} \cdot 2(2)\frac{dx}{dt}$

$6 = -3 \cdot \frac{1}{64} \cdot 4\frac{dx}{dt}$

$\frac{64}{-12} \cdot 6 = \frac{-12}{64}\frac{dx}{dt}$

$\boxed{-32 = \frac{dx}{dt}}$ (B)

④ $f' = 0$
critical pt
$f'' = +$
concave up
(B) $\boxed{\text{rel min}}$

⑤ 
(D) $\boxed{\text{inflection pt}}$

⑥ $f'(x) = 4x^3 + 3x^2$
$f''(x) = 12x^2 + 6x$
$= 6x(2x+1)$
$x = 0, -\frac{1}{2}$

-1	$-\frac{1}{2}$	-.25	0	+1
12-6		-.75		18
6 +		−		+

(A) $\boxed{(0,0)\ \ (-\tfrac{1}{2}, -\tfrac{1}{16})}$

⑦ $\frac{\infty}{\infty} \rightarrow \lim \frac{2x}{1/x} = \lim 2x^2$

(C) ∞

8) $\lim_{x\to 1} \frac{\ln x}{x-1} = \frac{0}{0} \Rightarrow \lim_{x\to 1} \frac{1/x}{1} = \boxed{1}$

9) $\lim_{x\to\infty} \frac{\ln x}{x^3} = \frac{\infty}{\infty} \Rightarrow \lim_{x\to\infty} \frac{1/x}{3x^2} = \lim_{x\to\infty} \frac{1}{3x^3} = 0$

☆ 10) $\frac{1}{16}$ A

11) $\frac{3e^{0/3} - 3 - 0}{0^2} = \frac{0}{0} \Rightarrow \lim_{x\to 0} \frac{3e^{x/3}\cdot\frac{1}{3} - 0 - 1}{2x} = \lim \frac{e^{x/3} - 1}{2x} = \frac{0}{0}$

$\lim \frac{e^{x/3}\cdot\frac{1}{3}}{2} = \lim \frac{1}{6} e^{x/3} = \frac{1}{6}$ C

☆ 12) $y' = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$L = \int_0^3 \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2}\, dx$ A

$= \int_0^3 \sqrt{1 + \frac{1}{4x}}\, dx$

D

☆ 13) $\frac{9x^2 + x - 1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$9x^2 + x - 1 = A(x^2 + x) + B(x+1) + Cx^2$

$9 = A + C \quad 1 = A + B \quad -1 = B$

$9 = 2 + C \quad 1 = A - 1$

$7 = C \quad 2 = A$

A

(14) $\dfrac{3x+4}{(x^2+4)(3-x)} = \dfrac{Ax+B}{x^2+4} + \dfrac{C}{3-x}$

$3x+4 = (Ax+B)(3-x) + C(x^2+4)$

$3x+4 = 3Ax + 3B - Ax^2 - Bx + Cx^2 + 4C$

$0 = C - A \quad 3 = 3A - B \quad 4 = 3B + 4C$

$A = C \quad 3(3 = 3C - B) \Rightarrow 9 = 9C - \cancel{3B}$

$A = 1 \quad 4 = 4C + 3B \quad 4 = 4C + \cancel{3B}$

$3 = 3 - B \quad 13 = 13C$

$B = 0 \quad 1 = C$

D

$\int \frac{1}{2}\left(\dfrac{2x}{x^2+4} + \dfrac{1}{3-x}\right) dx$

$\frac{1}{2}\ln|x^2+4| - \ln|3-x| + C$

$\ln\left|\dfrac{\sqrt{x^2+4}}{3-x}\right| + C$ (D)

(15) $u = x \quad dv = e^{2x}dx$

$du = 1\,dx \quad v = \frac{1}{2}e^{2x}$

$\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$

$\frac{1}{2}xe^{2x} - \frac{1}{2}\left[\frac{1}{2}e^{2x}\right] + C$

$\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

$\frac{2}{4}xe^{2x} - \frac{1}{4}e^{2x} + C$

$\frac{e^{2x}}{4}(2x-1) + C$ (C)

(16) $u = x \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$-x\cos x - \int -\cos x\,dx$

$-x\cos x + \sin x + C$ (D)

(17) $\int_1^3 \sqrt{(2t)^2 + (4t)^2}\,dt = \int_1^3 \sqrt{20t^2}\,dt$

$\sqrt{20}\int_1^3 t\,dt = \sqrt{20}\left[\frac{1}{2}t^2\right]_1^3$

$\sqrt{20}\left[\frac{1}{2}9 - \frac{1}{2}\right]$

$4\sqrt{20} = 4\cdot 2\sqrt{5} = 8\sqrt{5}$ (D)

(18) $\frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta}{-2\sin\theta}$

$$\frac{\frac{d}{d\theta}\left[\frac{\cos\theta}{-2\sin\theta}\right]}{\cancel{\theta}\ -2\sin\theta} = \frac{\frac{(-2\sin\theta)(-\sin\theta) - (\cos\theta)(-2\cos\theta)}{(-2\sin\theta)^2}}{-2\sin\theta}$$

$$\frac{2\sin^2\theta + 2\cos^2\theta}{4\sin^2\theta} \cdot \frac{1}{-2\sin\theta} = \frac{2}{-8\sin^3\theta}$$

$= -\frac{1}{4}\csc^3\theta$ (A)

(19) $\frac{dy}{dx} = \frac{2t+1}{t}$

$$\frac{\frac{d}{dt}\left[\frac{2t+1}{t}\right]}{t} = \frac{t(2) - (2t+1)1}{t^2} \cdot \frac{1}{t}$$

$$= \frac{2t - 2t - 1}{t^3} = \frac{-1}{t^3}$$ (B)

$(t-1)^{1/2}$

20) $\dfrac{\frac{1}{2}(t-1)^{-1/2}}{2t} = \dfrac{1}{2\sqrt{t-1}} \cdot \dfrac{1}{2t} = \dfrac{1}{4t\sqrt{t-1}}$ A

21) $\dfrac{3(t-1)^2}{\frac{1}{2}t^{-1/2}} = \dfrac{3(t-1)^2}{1/2\sqrt{t}} = 6\sqrt{t}(t-1)^2$ D

22) $\dfrac{dy}{dx} = \dfrac{2t}{2} = 1 = m$ $x=2$ $y=6$

$y-6 = 1(x-2)$

$y-6 = x-2$

$y = x+4$ C

23) $\dfrac{dy}{dx} = \dfrac{2t}{3} = \dfrac{2}{3} = m$ $x=2$ $y=1$

$y-1 = \frac{2}{3}(x-2)$

$y-1 = \frac{2}{3}x - \frac{4}{3}$

$3y-3 = 2x-4$

$2x-3y-1=0$ A

24) $y = 2t+1$ $x = 3t^2$ $4x = 3y^2-6y+3$

$y-1 = 2t$ $x = 3\left(\frac{y-1}{2}\right)^2$ $3y^2-4x-6y+3=0$

$\frac{y-1}{2} = t$ D

$x = 3\left(\frac{y^2-2y+1}{4}\right)$

(25) $x = 2\cos\theta \qquad y = \dfrac{x^2}{4}$

$\dfrac{x}{2} = \cos\theta$

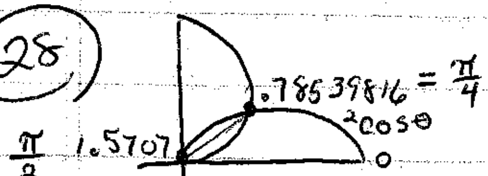
$\dfrac{x^2}{4} = \cos^2\theta \qquad 4y = x^2$ (D)

(26) $x = -4\cos\frac{\pi}{6} = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$y = -4\sin\frac{\pi}{6} = -4\left(\frac{1}{2}\right) = -2$ (A)

(27) $\dfrac{1}{2}\displaystyle\int_0^{2\pi}(1+\cos)^2\,d\theta = \dfrac{3\pi}{2}$ (C)

(28)



$2\cos\theta = 2\sin\theta$

$\cos\theta = \sin\theta$

$\theta = \pi/4$

$$\frac{1}{2}\int_{\frac{\pi}{4}}^{\pi/2}\left[2\cos\theta\right]^2 d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}}\left[2\sin\theta\right]^2 d\theta$$

$$2\left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos^2\theta\,d\theta + \int_0^{\frac{\pi}{4}}\sin^2\theta\,d\theta\right]$$

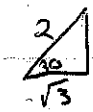
$$2\left[.142699081 7 + .142699081 7\right]$$

$.570796368 = \frac{\pi}{2} - 1$ (B)

(29) $.52359878$ $\frac{1}{2}\int_0^{\pi}\left[3\cos(3\theta)\right]^2 d\theta$

$\frac{9}{2}\int_0^{\pi}\cos^2(3\theta)\,d\theta = \frac{9}{2}\cdot\frac{\pi}{2} = \frac{9\pi}{4}$ (C)

(30) $\cancel{\frac{dy}{dx} = \frac{(2-3\sin\theta)\cos\theta + (-3\cos\theta)\sin\theta}{-(2-3\sin\theta)\sin\theta + (-3\cos\theta)\cos\theta}}$ $\theta = \frac{\pi}{6} = 30^\circ$



$\frac{d(r\sin\theta)}{d(r\cos\theta)}$

$\cancel{\frac{\left[2-3\left(\frac{1}{2}\right)\right]\frac{\sqrt{3}}{2} - 3\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}}{-\left[2-\frac{3}{2}\right]\frac{1}{2} - 3\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}} = \frac{+\frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \frac{3\sqrt{3}}{4}}{-\frac{1}{2}\cdot\frac{1}{2} - \frac{9}{4}}}$

(B)

$\cancel{= \frac{\frac{\sqrt{3}}{4} - \frac{3\sqrt{3}}{4}}{-\frac{1}{4} - \frac{9}{4}} = \frac{-2\sqrt{3}}{4}\cdot\frac{-4}{10} = \frac{\sqrt{3}}{5}}$ ~~E~~

$-\frac{10}{4}$

$\frac{(2\cos 3\theta)(\cos\theta) + (3\cdot 2\sin 3\theta)\sin\theta}{-(2\cos 3\theta)(\sin\theta) + (3\cdot 2\sin 3\theta)\cos\theta} = \frac{2\cos\frac{\pi}{2}\cos\frac{\pi}{6} + 6\sin\frac{\pi}{2}\sin\frac{\pi}{6}}{-2\cos\frac{\pi}{2}\sin\frac{\pi}{6} + 6\sin\frac{\pi}{2}\cos\frac{\pi}{6}}$

31 0 (A)

$\frac{2\cdot 0\cdot\frac{\sqrt{3}}{2} + 6(1)\frac{1}{2}}{-2\cdot 0\cdot\frac{1}{2} + 6(1)\frac{\sqrt{3}}{2}} = \frac{0+3}{-0+3\sqrt{3}} = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

32) $\lim_{n\to\infty}\frac{(n+1)^7}{7^{n+1}}\cdot\frac{7^n}{n^7} = \lim_{n\to\infty}\frac{(n+1)^7}{7n^7} = \frac{1}{7} < 1$ Conv A

33) lim n^{th} term $= \frac{1}{3} \neq 0$ Div D

34) $\frac{(n+1)!}{(n+1+5)!}\cdot\frac{\cancel{(n+5)!}}{\cancel{n!}} = \frac{n+1}{n+6} = 1$ inconclusive D

$(n+1)!$ = $(n+1)\cancel{n!}$

$(n+1+5)!$ = $(n+6)!$ = $(n+6)\cancel{(n+5)!}$

35) $\sqrt[n]{(\ )^n} = \lim \dfrac{2n-1}{3n+5} = \dfrac{2}{3}$ converge Root test A

36) $f(x) = \ln(x^2+4)$ $\quad f(0) = \ln 4$

$f'(x) = \frac{1}{x^2+4} \cdot 2x$ $\quad f'(0) = \frac{0}{4} = 0$

$f''(x) = \dfrac{(x^2+4)2 - 2x(2x)}{(x^2+4)^2}$ $\quad f''(0) = \dfrac{8-0}{16} = \dfrac{1}{2}$

$\ln 4 + 0x + \frac{1}{2}x^2 \cdot \frac{1}{2!}$

$\ln 4 + \frac{1}{4}x^2$ D

37) $f(x) = e^{3x}$ $\quad f(1) = e^3$

$f'(x) = 3e^{3x}$ $\quad f'(1) = 3e^3$

$f''(x) = 9e^{3x}$ $\quad f''(1) = 9e^3$

$f'''(x) = 27e^{3x}$ $\quad f'''(1) = 27e^3$

$e^3 + 3e^3(x-1) + \dfrac{9e^3}{\underset{2!}{2}}(x-1)^2 + \dfrac{\overset{9}{\cancel{27}}e^3(x-1)^3}{{}_2\cancel{6}}$ $\quad 3! \ 3\cdot2\cdot1$ D

38) A) $\lim \to 1/3 \neq 0$ Div B) Con p series $6>1$ C) Con Geo series $1/10 < 1$ D)

$\dfrac{n!/n!}{3n!/n! - 1/n!} = \dfrac{1}{3 - 1/n!}$

$\dfrac{n+1}{2^{n+1}} \cdot \dfrac{2^n}{n} = \dfrac{n+1}{2n} = \dfrac{1}{2} < 1$

Con Ratio Test

39) A) B) C) Div Geo Series $\frac{3}{2} > 1$ D) $\frac{1}{n^{1/2}}$ Div p series $\frac{1}{2} < 1$

$\dfrac{2^{n+1}}{(n+1+1)!} \cdot \dfrac{(n+1)!}{2^n}$

$\dfrac{2}{n+1}$ $\lim \to 0 < 1$

Con Ratio Test

40) $f(x) = \ln(x-2)$ $\qquad f(3) = \ln 1 = 0$

$f'(x) = \frac{1}{x-2} = (x-2)^{-1}$ $\qquad f(3) = 1$

$f''(x) = -1(x-2)^{-2}$ $\qquad f(3) = -1$ $\qquad (x-3) - \frac{(x-3)^2}{2} + \frac{(x-3)^3}{3} - \frac{(x-3)^4}{4}$

$f'''(x) = +2(x-2)^{-3}$ $\qquad f(3) = 2$

$f^4(x) = -6(x-2)^{-4}$ $\qquad f(3) = -6$

$$0 + 1(x-3) - \frac{1(x-3)^2}{\underset{2}{2!}} + \frac{\cancel{2}^{1}(x-3)^3}{\underset{\cancel{6}\,3}{3!}} - \frac{\cancel{6}^{1}(x-3)^4}{\underset{\cancel{24}\,4}{4!}}$$ E

41) $\sqrt[n]{\frac{1}{(\ln n)^n}} = \lim \frac{1}{\ln n} = \frac{1}{\infty} = 0 < 1$ Con C

42) $$\frac{\overset{(n+1)\cancel{n!}}{(n+1)!}}{\underset{\cancel{3^n}\,3}{3^{n+1}}} \cdot \frac{\cancel{3^n}}{\cancel{n!}} = \frac{n+1}{3} = \infty > 1$$ ~~Converge~~ B

Diverges