1) $(-\sqrt{3}, 1)$ 2) $(2, 5\pi/6)$

3) $y = 3$ 4) $r = 3\sin\theta$

5) Vert: $x = \frac{3\sqrt{3}}{4}, x = \frac{-3\sqrt{3}}{4}$ Hor: $y = 2, y = -1/4$

6) π 7) .544 8) 4.914

9) no tangents @ pole (cusp)

10) 24 11) π 12) $y - \frac{\sqrt{3}}{6} = -1.347\left(x + \frac{1}{2}\right)$

13) 33.351 14) 9.688

15) 9.352 16) $\theta = \pi/2$
$\theta = 3\pi/2$

17) r is decreasing, curve moving away from pole.

This answer key has answers to a longer review.

Polar Review Sheet

+	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	$\frac{\sqrt{0}}{2}$	$\sqrt{1}$	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{4}$
cos	$\frac{\sqrt{4}}{2}$	$\sqrt{3}$	$\sqrt{2}$	$\sqrt{1}$	$\sqrt{0}$

1) a) $x = 5\cos(3\pi/4) = 5\cdot\frac{\sqrt{2}}{2}$

$y = 5\sin(3\pi/4) = \frac{5\sqrt{2}}{2}$

$\left(-\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$

b) $x = -2\cos(11\pi/6) = -2\cdot\sqrt{3}/2 = -\sqrt{3}$

$y = -2\sin(11\pi/6) = -2\cdot -1/2 = 1$

$(-\sqrt{3}, 1)$

c) $x = \cos\pi = -1$

$y = \sin\pi = 0$

$(-1, 0)$

2) a) $\sqrt{(3\sqrt{2})^2 + (3\sqrt{2})^2} = r = 6$

$\tan\theta = 1$

$\theta = \pi/4$

$(6, \pi/4)$

b) $\sqrt{\sqrt{3}^2 + 1^2} = r = 2$

$\tan\theta = -1/\sqrt{3}$

$\theta = \pi/6 \to 5\pi/6$

$(2, 5\pi/6)$

c) $\sqrt{1^2 + (-2)^2} = \sqrt{5} = r$

$\tan\theta = \frac{-2}{1}$

$\theta = -1.107$

$(\sqrt{5}, -1.107)$

3) a) $x^2 + y^2 = 100$

b) $\tan 3\pi/4 = y/x$

$-x = y$

c) $r = \frac{3}{\sin\theta}$

$r\sin\theta = 3$

$y = 3$

4) a) $r^2 - 3r\sin\theta = 0$

$r^2 = 3r\sin\theta$

$r = 3\sin\theta$

b) $r\cos\theta = 10$

$r = 10\sec\theta$

c) $r\cos\theta \cdot r\sin\theta = 4$

$r^2 = 4\csc\theta\sec\theta$

5) $r = \sin(2\theta)$

$x = \sin(2\theta)\cos\theta$

$y = \sin(2\theta)\sin\theta$

$\frac{dy}{dx} = \frac{\sin(2\theta)\cos\theta + \sin\theta\cdot 2\cos(2\theta)}{-\sin(2\theta)\cdot\sin\theta + \cos\theta\cdot 2\cos(2\theta)}$

@ $\pi/3$ $\frac{dy}{dx} = .346$ @ $\pi/4$ $\frac{dy}{dx} = -1$

6) $r = 1 + \sin\theta$

vert $x = (1+\sin\theta)\cos\theta$

$= \cos\theta + \sin\theta\cos\theta$

$\frac{dx}{dt} = -\sin\theta + -\sin^2\theta + \cos^2\theta$

$= -\sin\theta - \sin^2\theta + 1 - \sin^2\theta$

$= 1 - \sin\theta - 2\sin^2\theta$

$0 = (1 - 2\sin\theta)(1 + \sin\theta)$

$\sin\theta = 1/2 \quad \sin\theta = -1$

$\theta = \pi/6, 5\pi/6, 3\pi/2$

$(3/2, \pi/6)(3/2, 5\pi/6)$

horiz $y = (1+\sin\theta)\sin\theta$

$= \sin\theta + \sin^2\theta$

$\frac{dy}{dt} = \cos\theta + 2\sin\theta\cos\theta = 0$

$\cos\theta(1 + 2\sin\theta) = 0$

$\cos\theta = 0 \quad \sin\theta = -1/2$

$\theta = \pi/2, 3\pi/2, 7\pi/6, 11\pi/6$

$(2, \pi/2)(1/2, 7\pi/6)(1/2, 11\pi/6)$

$x = r\cos\theta$ $\pi/6$

$x = 3/2\cos$

$x = 3/2 \cdot \sqrt{3}/2$

$x = \frac{3\sqrt{3}}{4}$

$x = -\frac{3\sqrt{3}}{4}$

$y = r\sin\theta$

$y = 2\sin\pi/2$

$y = 2$

$y = 1/2 \cdot -1/2$

$y = -1/4$

⑦ $r=\sin\theta$ $[0,\pi]$

$$\int_0^{\pi}\sqrt{(\sin\theta)^2+(\cos\theta)^2}\,d\theta = \boxed{\pi}$$

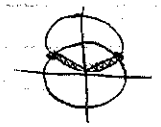
⑧ $r=1-2\sin\theta=0$ inner loop

$\sin\theta=\frac{1}{2}$

$\theta=\pi/6,\ 5\pi/6$

$$\frac{1}{2}\int_{\pi/6}^{5\pi/6}(1-2\sin\theta)^2\,d\theta \neq \boxed{.544}$$

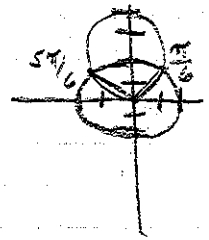
⑨ $r=4\sin\theta$ $r=2$



$4\sin\theta=2$

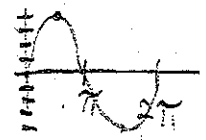
$\sin\theta=\frac{1}{2}$

$\theta=\pi/6,\ 5\pi/6$



$$2\cdot\frac{1}{2}\int_0^{\pi/6}(4\sin\theta)^2\,d\theta+\frac{1}{2}\int_{\pi/6}^{5\pi/6}2^2\,d\theta$$

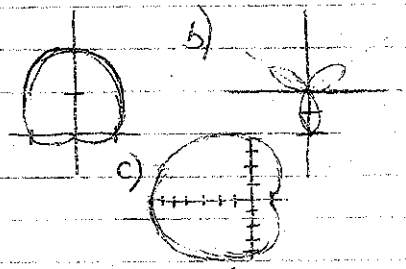
$$.725+4.189=\boxed{4.914}$$



⑩ $r=\sin\theta$ polar axis

$$2\pi\int_0^{\pi}\sin^2\theta\sqrt{\sin^2\theta+\cos^2\theta}\,d\theta=\boxed{\pi^2}$$

① a) cardioid
b) rose w/3 petals
c) limaçon



⑫ $r=1-2\cos\theta$

vert $x=(1-2\cos\theta)\cos\theta$

$=\cos\theta-2\cos^2\theta$

$\frac{dx}{d\theta}=-\sin\theta+4\cos\theta\sin\theta=0$

$=\sin\theta(-1+4\cos\theta)=0$

$\sin\theta=0$ $\cos\theta=\frac{1}{4}$

$\theta=0,\pi,1.318,4.965$

$\boxed{(-1,0)(3,\pi)(.5,1.318)(.5,4.965)}$

horiz $y=(1-2\cos\theta)\sin\theta$

$=\sin\theta-2\cos\theta\sin\theta$

$\frac{dy}{d\theta}=\cos\theta-2\cos^2\theta+2\sin^2\theta$

$=\cos\theta-2\cos^2\theta+2(1-\cos^2\theta)$

$=\cos\theta-2\cos^2\theta+2-2\cos^2\theta$

$0=2+\cos\theta-4\cos^2\theta$

$\theta=1.047,\pi/2,3\pi/2,5.236$

$\boxed{(0,1.047)(1,\pi/2)(1,3\pi/2)(0,5.236)}$

⑬ $r=6\sin(2\theta)$

$$\frac{1}{4}\cdot\frac{1}{2}\int_0^{2\pi}(6\sin(2\theta))^2\,d\theta=\boxed{14.137}$$

⑭ $5\sin\theta=2+\sin\theta$

$4\sin\theta=2$

$\sin\theta=\frac{1}{2}$

$\theta=\pi/6,\ 5\pi/6$

$\boxed{(3/2,\pi/6)(3/2,5\pi/6)}$

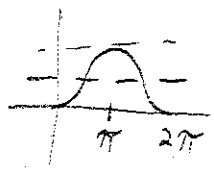
⑮ $1-\cos\theta=0$

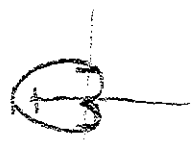
$\cos\theta=1$

$\theta=0$

$r'=\sin\theta=0$

no tangents @ pole (cusp)





(16) $r = 3(1+\cos\theta)$ $\quad r' = 3(-\sin\theta)$

$$\int_0^{2\pi} \sqrt{(3+3\cos\theta)^2 + (3\sin\theta)^2}\, d\theta = \boxed{24}$$

(17) $r = -2\sin\theta$

$$\frac{1}{2}\int_0^{\pi} (-2\sin\theta)^2\, d\theta = \boxed{\pi}$$

(18) $r = 2-2\sin\theta$ $\quad \theta = \pi/2$

$$2\pi \int_{3\pi/2}^{5\pi/2} (2-2\sin\theta)\cos\theta \sqrt{(2-2\sin\theta)^2 + (-2\cos\theta)^2}\, d\theta$$

$$= \boxed{45.587}$$

(19) $r = \cos(3\theta)$

$$\frac{1}{2}\int_0^{\pi} (\cos(3\theta))^2\, d\theta = \boxed{.785} \quad \frac{\pi}{4}$$

(20) $r = \tan\theta$

$x = \frac{\sin\theta}{\cos\theta}\cos\theta = \sin\theta$ $\quad \frac{dy}{dx} = \frac{\tan\theta\cos\theta + \sin\theta\sec^2\theta}{\cos\theta}$

$y = \tan\theta\sin\theta$ $\quad @ 11\pi/6 \ \frac{dy}{dx} = -1.347$

$x = -\frac{1}{2}$ $\quad y = \frac{\sqrt{3}}{6}$

$$\boxed{y - \frac{\sqrt{3}}{6} = -1.347\left(x + \frac{1}{2}\right)}$$

(21) $r = 3-2\sin\theta$ $\quad r = 4\cos\theta$

$3 - 2\sin\theta = 4\cos\theta$ *calculator (Sorry)

$\theta = 1.299, 5.911$

$$\frac{1}{2}\int_0^{1.299} (3-2\sin(\theta))^2\, d\theta + \frac{1}{2}\int_{1.299}^{5.911} (4\cos(\theta))^2\, d\theta$$

$$+ \frac{1}{2}\int_{5.911}^{2\pi} (3-2\sin(\theta))^2\, d\theta = 3911 + 16.059 + 1.889 = \boxed{21.859}$$

(22) $2 - 3\cos\theta = \cos\theta$

$2 = 4\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \pi/3, \ 5\pi/3$

$$\boxed{\left(\tfrac{1}{2}, \tfrac{\pi}{3}\right)\left(\tfrac{1}{2}, \tfrac{5\pi}{3}\right)}$$

(23) $r = 1-\sin\theta$ $\quad [0, \pi]$

$$\frac{1}{2}\int_0^{\pi} (1-\sin\theta)^2\, d\theta = \boxed{.356}$$

(24) $r = 2+4\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = 7\pi/6, \ 11\pi/6, \ 19\pi/6$

inner, outer

$$\frac{1}{2}\int_{11\pi/6}^{19\pi/6} (2+4\sin\theta)^2\, d\theta - \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (2+4\sin\theta)^2\, d\theta$$

$$35.525 - 2.174 = \boxed{33.357}$$

(25) $r = -\sin(5\theta) = 0 \qquad r' = -5\cos(5\theta) \neq 0$

$5\theta = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

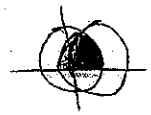
$\theta = 0, \pi/5, 2\pi/5, 3\pi/5, 4\pi/5, \pi$

(26) $r = 4\cos(2\theta) \qquad r' = -8\sin(2\theta)$

$$\frac{1}{4}\int_0^{2\pi} \sqrt{(4\cos(2\theta))^2 + (8\sin(2\theta))^2}\, d\theta$$

$= 9.688$

(27) $r = 2 + \cos\theta \qquad r = 2$



$2 + \cos\theta = 2$

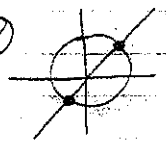
$\cos\theta = 0$

$\theta = \pi/2, 3\pi/2$

$$2 \cdot \frac{1}{2}\int_0^{\pi/2} (2)^2 + 2\,\frac{1}{2}\int_{\pi/2}^{\pi} \cdot (2+\cos\theta)^2\, d\theta$$

$2\pi + 3.069 = 9.352$

(28)



$(2, \pi/4)\ (-2, \pi/4)$

(29) $r = 3\cos\theta = 0 \qquad r = -3\sin\theta \neq 0$

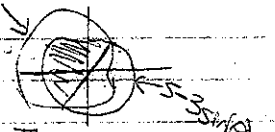
$\theta = \pi/2, 3\pi/2$

(30) $r = 5 - 3\sin\theta \qquad r = 5 - 3\cos\theta$

$5 - 3\sin\theta = 5 - 3\cos\theta$

$\sin\theta = \cos\theta$

$\theta = \pi/4, 5\pi/4$



$$\frac{1}{2}\int_{\pi/4}^{5\pi/4} (5-3\sin\theta)^2\, d\theta + \frac{1}{2}\int_0^{\pi/4} (5-3\cos\theta)^2\, d\theta + \frac{1}{2}\int_{5\pi/4}^{2\pi} (5-3\cos\theta)^2\, d\theta$$

$25.125 + 2.103 + 23.022 = 50.25$

(31) $\dfrac{dr}{d\theta} = -\sin\theta \qquad \left.\dfrac{dr}{d\theta}\right|_{\theta=\frac{3\pi}{4}} = -\dfrac{\sqrt{2}}{2}$

So r is decreasing

$r\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

If both r and $\frac{dr}{d\theta}$ are negative, the curve is moving away from the origin.