Serious (Series) Review

1. Let f be a function defined by $f(x)=\dfrac{1}{x-1}$.
   a. Write the first four terms and the general term of the Taylor series expansion of $f(x)$ about $x=2$.
   b. Use the result from part a to find the first four terms and the general term of the series expansion about $x=2$ for $\ln|x-1|$.

2. Let f be a function defined by $f(x)=e^{-2x^2}$.
   a. Find the first four nonzero terms and the general term of the power series for $f(x)$ about $x=0$.
   b. Find the interval of convergence of the power series for $f(x)$ about $x=0$. Show the analysis that leads to your conclusion.
   c. Calculate the error in approximating e^{-2} by using the fifth-degree Taylor polynomial about x = 0 for $f(x)$.

3. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(1)=3,\quad f'(1)=-2,\quad f''(1)=2,\quad f'''(1)=4$.
   a. Write the second degree Taylor polynomial for f about $x=1$ and use it to approximate $f(0.7)$.
   b. Write the third degree Taylor polynomial for f about $x=1$ and use it to approximate $f(1.2)$.
   c. Write the second degree Taylor polynomial for f', the derivative of f, about $x=1$ and use it to approximate $f'(1.2)$.

4. Find the Taylor series for $f(x)=e^{3x}$ centered about 1.

5. Find a power series for $f(x)=\dfrac{2}{(x-3)^2}$ and give its interval of convergence.

Find the interval of convergence for the following power series.

6. $\displaystyle\sum_{n=0}^{\infty}\frac{(3x)^n}{(2n)!}$

7. $\displaystyle\sum_{n=0}^{\infty}\frac{(x-2)^{n+1}}{(n+1)4^{n+1}}$

8. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$