Chapter 7 Derivatives Review

1. Evaluate the limit.

   a. $\lim_{x\to 0}\frac{3x-\sin x}{x}$ b. $\lim_{x\to\infty}\frac{2^x}{x^2}$

2. Calculate y'.

   a. $xy^4 + x^2y = x + 3y$ b. $x^2cosy + sin2y = xy$ c. $\sin(xy) = x^2 - y$

   d. $xtany = y - 1$ e. $y = 2^{-x^2}$ $f.\ y = xtan^{-1}x$

   g. $y = log_{10}(1 + x^2)$ h. $y = (cosx)^x$ i. $y = \tan^{-1}(\sin^{-1}\sqrt{x})$

3. Find an equation of the tangent to the curve at the given point.

   a. $y = (2 + x)e^{-x}$, (0,2) b. $y = xlnx$, (e, e)

   c. $x^2 + 4xy - y^2 = 13$, (2,1)

4. If a point moves on the curve $x^2 + y^2 = 25$, find $\frac{d^2y}{dx^2}$ at (0,5).

5. Let $f(x) = \frac{1}{4}x^3 + x - 1$, and let f^{-1} denote the inverse of f. Then $(f^{-1})'(3)$ is equal to ?

6. If f(4) = 5 and f'(4) = 2/3, then calculate $(f^{-1})'(5)$.

5) $\frac{d}{dx}\left(x = \frac{1}{4}y^3 + y - 1\right)$

$1 = \frac{3}{4}y^2\frac{dy}{dx} + \frac{dy}{dx}$

$1 = \left(\frac{3}{4}y^2 + 1\right)\frac{dy}{dx}$

$\frac{1}{\frac{3}{4}y^2 + 1} = \frac{dy}{dx}$

$3 = \frac{1}{4}y^3 + y - 1$

$y = 2$

$3 = \frac{1}{4}(8) + 2 - 1$

$(f^{-1})'(3) = \frac{1}{\frac{3}{4}(2)^2 + 1}$

$x = 3$, $y = 2$

$= \frac{1}{3+1} = \boxed{\frac{1}{4}}$

6) $(f^{-1})'(5) = \frac{1}{f'(4)} = \frac{1}{2/3} = \boxed{\frac{3}{2}}$

$(f^{-1})'(3) = \frac{1}{f'(2)}$