Series Review Sheet

1. p-series
$p = 1 \leq 1$
$\therefore$ diverges

2. $\sum \frac{1}{5^n}$ geo series
$r = \frac{1}{5} < 1 \therefore$ converges
$\frac{1}{5^n+7} < \frac{1}{5^n}$
$\therefore \sum \frac{1}{5^n+7}$ also converges by DCT

3. $\lim_{n\to\infty} \frac{4n}{12n+5} = \frac{4}{12} \neq 0$
$\therefore$ diverges by n^{th} term test

4. geo. series
$r = 2 > 1$
$\therefore$ diverges

5. ratio test
$$\lim_{n\to\infty} \left| \frac{\frac{3^{n+1}}{(n+1)^3}}{3^n/n^3} \right| = 3$$
$\lim > 1 \therefore$ diverges

6. p series
$p = 5/2 > 1$
$\therefore$ converges

7. p series
$p = 2 > 1$
$\therefore$ converges

8. root test
$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{n^n}{(3n+3)^n}\right|} = \frac{1}{3}$$
$\lim < 1 \therefore$ converges

9. $\sum 1/n^{1/2}$ p series
$p = 1/2 < 1 \therefore$ diverges
$$\lim_{n\to\infty} \left| \frac{\frac{1}{\sqrt{n}+5}}{1/\sqrt{n}} \right| = 1 \neq 0$$
$\therefore \frac{1}{\sqrt{n}+5}$ also diverges by L

10) $\lim_{n\to\infty} 4 = 4 \neq 0$
$\therefore$ diverges by n^{th} term test

11) $\sum \frac{n}{n^3} = \sum \frac{1}{n^2}$ converges
$$\lim_{n\to\infty} \frac{n/2n^3+2}{1/n^2} = \frac{1}{2}$$
$\lim \neq 0 \therefore$
$\sum \frac{n}{2n^3+2}$ also converges by LCT

12) geo series
$r = \frac{2}{3} < 1$
$\therefore$ converges to 3
$$S = \frac{1}{1-2/3} = 3$$

13) ratio test

$$\lim_{n\to\infty}\left|\frac{2^{n+1}/(n+1)!}{2^n/n!}\right| = 0$$

$\lim < 1$ $\therefore$ converges

14) $\sum \frac{n}{n^2} = \sum \frac{1}{n}$ diverges

$$\lim_{n\to\infty}\left|\frac{\frac{n}{n^2+1}}{1/n}\right| = 1 \neq 0$$

$\therefore \sum \frac{n}{n^2+1}$ also diverges by LCT

15) $\lim_{n\to\infty} \frac{n!}{3n!+1} = \frac{1}{3}$

$\lim \neq 0$

$\therefore$ diverges by n^{th} term test

16. $\left(1+\frac{1}{2}+\frac{1}{3}+\cdots\right)-\left(\frac{1}{2}+\frac{1}{3}+\cdots\right)$

$= 1 - \lim_{n\to\infty} \frac{1}{n+1} = 1$

$\therefore$ telescoping series converges to $\underline{1}$

17. geo series

$r = \frac{1}{2} < 1$

$\therefore$ converges to 6

$S = \frac{3}{1-1/2} = 6$

18. geo series

$r = 1/10 < 1$

$\therefore$ converges to $\frac{.4}{1-1/10} = 4/9$

19. Alternating Series

$\lim_{n\to\infty} \frac{1}{n\ln n} = 0$, $\frac{1}{(n+1)\ln(n+1)} < \frac{1}{n\ln n}$

$\therefore$ converges conditionally

$$\int_1^\infty \frac{1}{x\ln x}\,dx$$

$\lim_{b\to\infty} \ln(\ln x)\Big]_1^b$

$\lim_{b\to\infty} \ln(\ln b) - \ln(\ln(1))$

$\infty - -\infty$

20. $\sum_{n=0}^{\infty} \frac{1}{3}\left(-\frac{1}{2}\right)^n$

geo series

$|r| = \frac{1}{2} < 1$

$\therefore$ converges absolutely to $\frac{1/3}{1--1/2} = 2/9$

21) $a_n = \frac{2^n}{n}$

$\lim_{n\to\infty} \frac{2^n}{n} = \lim_{n\to\infty} \frac{2^n\ln 2}{1} = \infty$

sequence diverges

22. $a_1 = 1$

$a_2 = .5 = \frac{1}{2}$

$a_3 = .17 = \frac{1}{6}$

$a_4 = .04 = \frac{1}{24}$

$a_5 = .008 = \frac{1}{120}$

$a_6 = .001 = \frac{1}{720}$

$a_7 = .0001 = \frac{1}{5040}$

$|S - S_6| < a_7$

$\left|S - \frac{-91}{144}\right| < \frac{1}{5040}$

$-\frac{1}{5040} < S + \frac{91}{144} < \frac{1}{5040}$

$-.6321 < S < -.6317$

$S_6 = -1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720}$

$S_6 = -.631944444$

$-\frac{91}{144}$

$\boxed{S = -.632}$