1. $\int \ln x\,dx = x\ln x - \int \cancel{x}\frac{1}{\cancel{x}}\,dx = x\ln x - x + C$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

2. $\int x e^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}\,dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$

$u = x \quad dv = e^{3x}dx$

$du = 1dx \quad v = \frac{1}{3}e^{3x}$

3. $\int x^3 \sin x\,dx = -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$

$u = x^3 \quad dv = \sin x\,dx$

$3x^2 \quad -\cos x$

$6x \quad -\sin x$

$6 \quad \cos x$

$0 \quad \sin x$

4. $\frac{x^2+2x-1}{2x^3+3x^2-2x} = \frac{x^2+2x-1}{x(2x-1)(x+2)} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2}$

$x^2+2x-1 = A(2x-1)(x+2) + Bx(x+2) + C(x)(2x-1)$

let $x = -2$ | let $x = 0$ | let $x = 1$

$-1 = C(-2)(-5)$ | $-1 = A(-1)(2)$ | $2 = A(1)(3) + 3B + C$

$-\frac{1}{10} = C$ | $\frac{1}{2} = A$ | $10\left(2 = \frac{3}{2} + 3B - \frac{1}{10}\right)$

$20 = 15 + 30B - 1$

$6 = 30B$

$\frac{1}{5} = B$

$\int \frac{\frac{1}{2}}{x} + \frac{\frac{1}{5}}{2x-1} + \frac{-1/10}{x+2}\,dx$

$= \frac{1}{2}\ln|x| + \frac{1}{2}\cdot\frac{1}{5}\int \frac{1\cdot 2}{2x-1}\,dx - \frac{1}{10}\ln|x+2|$

$u = 2x-1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= \frac{1}{2}\ln|x| + \frac{1}{10}\ln|2x-1| - \frac{1}{10}\ln|x+2| + C$

5. $$x^3-x^2-x+1 \overline{\big) x^4-2x^2+4x+1} \quad = x+1+\frac{4x}{x^3-x^2-x+1}$$

$$\begin{array}{r} x^4-2x^2+4x+1 \\ -(x^4-x^3-x^2+x) \\ \hline x^3-x^2+3x+1 \\ -(x^3-x^2-x+1) \\ \hline 4x \end{array}$$

$$x^3-x^2-x+1$$
$$x^2(x-1)-1(x-1)$$
$$(x^2-1)(x-1)$$
$$(x+1)(x-1)(x-1)$$

$$\frac{4x}{x^3-x^2-x+1}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$

$$4x=A(x-1)^2+B(x-1)(x+1)+C(x+1)$$

let $x=1$ | let $x=-1$ | let $x=0$

$4=C\cdot 2$ | $-4=A\cdot 4$ | $0=A-B+C$

$2=C$ | $-1=A$ | $0=-1-B+2$

$B=1$

$$\int \frac{2}{(x-1)^2}dx$$
$$2\int (x-1)^{-2}dx$$
$$2\frac{(x-1)^{-1}}{-1}$$

$$\int x+1-\frac{1}{x+1}+\frac{1}{x-1}+\frac{2}{(x-1)^2}\,dx$$

$$=\frac{1}{2}x^2+x-\ln|x+1|+\ln|x-1|\frac{-2}{x-1}+C$$

6. $$\int e^{2x}\sin(3x)dx=-\frac{1}{3}e^{2x}\cos(3x)-\int -\frac{2}{3}e^{2x}\cos(3x)dx$$

$u=e^{2x}$ $\quad dv=\sin(3x)dx$

$du=2e^{2x}dx$ $\quad v=-\frac{1}{3}\cos(3x)$

$$-\frac{1}{3}e^{2x}\cos(3x)+\frac{2}{3}\int e^{2x}\cos(3x)\,dx$$

$u=e^{2x}$ $\quad dv=\cos(3x)dx$

$du=2e^{2x}dx$ $\quad v=\frac{1}{3}\sin(3x)$

$$\int e^{2x}\sin(3x)dx=-\frac{1}{3}e^{2x}\cos(3x)+\frac{2}{3}\left[\frac{1}{3}e^{2x}\sin(3x)-\int\frac{2}{3}e^{2x}\sin(3x)dx\right]$$

$$\int e^{2x}\sin(3x)dx=-\frac{1}{3}e^{2x}\cos(3x)+\frac{2}{9}e^{2x}\sin(3x)-\frac{4}{9}\int e^{2x}\sin(3x)dx$$

$$\frac{13}{9}\int e^{2x}\sin(3x)dx=-\frac{1}{3}e^{2x}\cos(3x)+\frac{2}{9}e^{2x}\sin(3x)$$

$$\int e^{2x}\sin(3x)dx=\frac{9}{13}\left[-\frac{1}{3}e^{2x}\cos(3x)+\frac{2}{9}e^{2x}\sin(3x)\right]+C$$