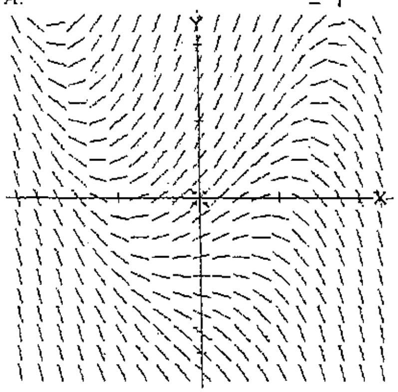
No Calculators!!!!!!! Name ______________________

Calculus Test
Diff Eq's: Part 1

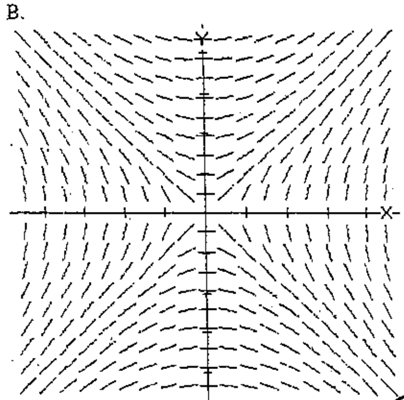
1. Match the differential equation with its slopefield.

C Vert. Pattern $\frac{dY}{dX} = X(e^{-X})$ D $\frac{dY}{dX} = \frac{X-Y}{X+Y}$ zero B $\frac{dY}{dX} = \frac{X}{Y}$ A $\frac{dY}{dX} = Y - X^2 + 1$

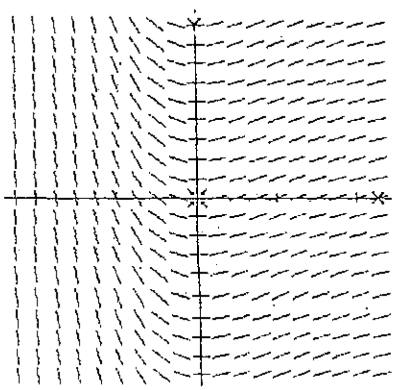
A.



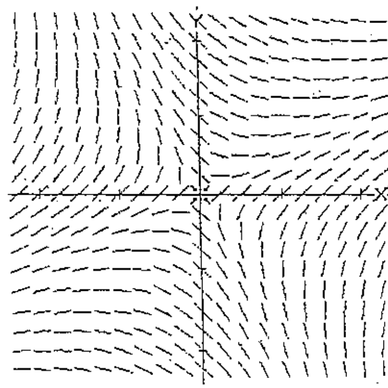
B.



C.



D.



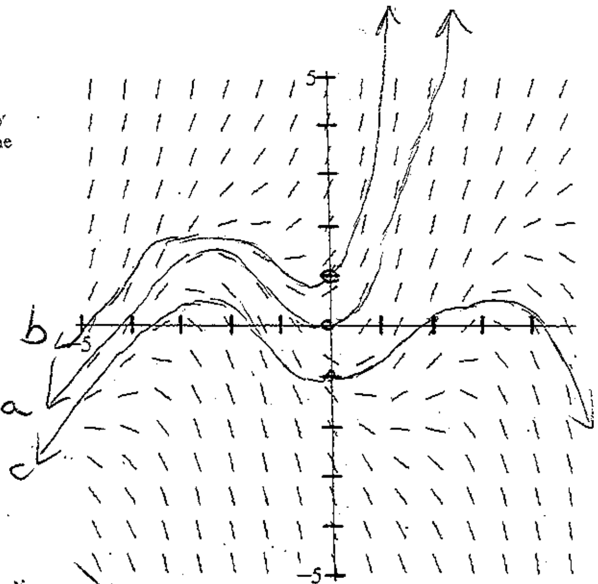
No Calculators!!!!!!!!!!!!!

2. On the slopefield, sketch and clearly label the solution path for each of the given initial conditions.

   a. $y(0) = 0$

   b. $y(0) = 1$

   c. $y(0) = -1$



$-2e^x\sin x - 2(e^x\cos x - e^x\sin x) + 2e^x\cos x = 0$

3. Verify that $y = e^x\cos x$ is a solution to the initial value problem below.

$y'' - 2y' + 2y = 0$, where $y(0) = 1$

$y' = e^x\cos x + e^x\sin x$

$y'' = \cancel{e^x\cos x} - e^x\sin x - e^x\sin x - \cancel{e^x\cos x}$

4. Consider the initial value problem: $\frac{dP}{dt} = 9t(P+t)$, where $P(0) = 2$

$P(1) \approx \frac{43}{3}$

   a. Using $\Delta t = \frac{1}{3}$; approximate $P(1)$. Show **ALL** steps below.

$P(1) = \frac{13}{3} + 9\left(\frac{2}{3}\right)\left(\frac{13}{3} + \frac{2}{3}\right)\left(\frac{1}{3}\right) = \frac{43}{3}$

$P\left(\frac{1}{3}\right) = 2 + 9(0)(2+0)\left(\frac{1}{3}\right) = 2$ $P\left(\frac{2}{3}\right) = 2 + 9\left(\frac{1}{3}\right)\left(2 + \frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{7}{3} + 2 = \frac{13}{3}$

   b. We know this answer is only an approximate. In light of that, would you expect the actual value of $P(1)$ to be greater than this approximate or less? Briefly explain your answer.

$P'' = 9(P+t) + 9t(P'+1)$

pos + pos(pos+1) = pos

concave up

so actual value is greater than approx

5. Solve the following initial value problem.

$\frac{dy}{dx} = (\sin x)\sqrt{2y+5}$, where $y(0) = 2$

$\frac{1}{2}\int 2(2y+5)^{-1/2}\,dy = \sin x\,dx$

$\cancel{2}\frac{1}{\cancel{2}}(2y+5)^{1/2} = -\cos x + C$

$3 = -1 + C$

$4 = C$

$(2y+5)^{1/2} = 4 - \cos x$

$\cancel{2y+5} = (4-\cos x)^2 - 5$
$\frac{(4-\cos x)^2 - 5}{2}$

$\boxed{y = \frac{(4-\cos x)^2 - 5}{2}}$

Exponential Growth and Decay $\frac{1}{2}\cancel{A} = \cancel{A}e^{k\,24,360}$

1. Radioactive Decay. Suppose 10 grams of the plutonium isotope PU-239 was released in the Chernobyl nuclear accident. How long will it take for the 10 grams to decay to 1 gram?

half-life → 24,360 yrs.

$m(t) = 10e^{\frac{1}{24,360}\ln(\frac{1}{2})t}$

$\frac{1}{10} = e^{\frac{1}{24360}\ln(\frac{1}{2})t}$

$\frac{24,360\ln(\frac{1}{10})}{\ln(\frac{1}{2})} = t$

$t = 80,922.168$ yrs

2. A population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day and 300 flies after day 4. How many flies were in the original population?

$y(t) = y_0 e^{kt}$

$100 = y_0 e^{2k} \Rightarrow \frac{100}{e^{2k}} = y_0$

$300 = y_0 e^{4k} \Rightarrow 300 = \frac{100}{e^{2k}} \cdot \frac{e^{4k}}{1}$

$300 = 100e^{2k}$

$3 = e^{2k}$

$\ln 3 = 2k$

$\frac{\ln 3}{2} = k$

$100 = y_0 e^{2(\frac{\ln 3}{2})}$

$\frac{100}{3} = y_0$ 3

$300 = y_0 e^{4(\frac{\ln 3}{2})}$

$300 = y_0 e^{2\ln 3^2}$

$\frac{300}{9} = y_0 3^2$

$y_0 \approx 33.3\overline{3}$ flies

3. 4 months after it stops advertising, a manufacturing company notices that its sales have dropped from 100,000 units per month to 80,000 units per month. If the sales follow an exponential pattern of decline, what will they be after another 2 months?

$y(t) = y_0 e^{kt}$

$80,000 = 100,000e^{k(4)}$

$\frac{4}{5} = e^{4k}$

$\ln(\frac{4}{5}) = 4k$

$\frac{1}{4}\ln(\frac{4}{5}) = k$

$y(6) = 100,000e^{\frac{1}{4}\ln(\frac{4}{5})6}$

$y(6) = 71,554.175$ units

4. An object is left in a room whose temp is kept at a constant 60°. If the object cools from 100° (T_0) to 90° ($T(10)$) in 10 minutes, how much longer will it take for its temp to decrease to 80°?

$T(t) = (T - T_s)e^{kt}$

$30 = 40e^{k(10)}$

$\frac{3}{4} = e^{10k}$

$\ln(.75) = 10k$

$\frac{1}{10}\ln(.75) = k$

$20 = 40e^{\frac{1}{10}\ln(\frac{3}{4})t}$

$.5 = e^{\frac{1}{10}\ln .75t}$

$\frac{10\ln .5 = \cancel{\ln .75}\,t}{\ln .75}$

$t = 24.094$ mins

-10

14.094 mins

Solve each differential equation for the given initial conditions. Find your answer in the Functions column; write the letter next to the function in the box at the bottom of the page that contains the number of that exercise.

Differential equation — **Initial condition** — **Functions**

1. $y^{-2}\frac{dy}{dx} = 4xy^2$ dx — $(0,1)$ — N $y = 4e^{\frac{x^2}{2}}$ 2

$-1y^{-1} = 2x^2 + C$

$\frac{1}{y} = -2x^2 + C$

2. $1 = xy\frac{dx}{dy}$ — $(0,4)$ $y = Ce^{\frac{x^2}{2}}$ — G $y = 2e^{x^2} - 2$ 3

$\frac{1}{y}dy = x\,dx$

$\ln y = \frac{1}{2}x^2 + C$

3. $\frac{dy}{dx} = 2xy + 4x$ — $(0,0)$ — A $y = \frac{1}{1-2x^2}$ 1

$2x(y+2)$

$\frac{1}{y+2}dy = 2x\,dx$

$\ln(y+2) = x^2 + C$

$y = Ce^{x^2} - 2$

4. $y\frac{dy}{dx} = (3x+1)$ dx — $(0,1)$ — E $y = \sqrt{2x^2 + 14}$ 5

$\frac{1}{2}y^2 = \frac{3}{2}x^2 + 2x + C$

5. $\frac{dy}{dx} = \frac{2x}{y}$ — $(1,4)$ — S $y = \sqrt{\ln x + 4}$ 6

$y\,dy = 2x\,dx$

$y = \sqrt{2x^2 + C}$ 4

6. $2\frac{dy}{dx} = \frac{1}{xy}$ — $(1,2)$ — L $y = \sqrt{3x^2 + 2x + 1}$ 4

$2y\,dy = \frac{1}{x}dx$

$y^2 = \ln x + C$ 4

Old mathematicians never die ... they just become

1	2	3	4	5	6
A	N	G	L	E	S