1994 - BC 3

3. A particle moves along the graph of $y = \cos x$ so that the x-component of acceleration is always 2. At time $t = 0$, the particle is at the point $(\pi, -1)$ and the velocity vector of the particle is (0,0).

(a) Find the x- and y- coordinates of the position of the particle in terms of t.

(b) Find the speed of the particle when its position is $(4, \cos 4)$.

a.) $\frac{d\left(dx/dt\right)}{dt} = 2 \Rightarrow \int d\left(dx/dt\right) = \int 2\,dt \Rightarrow \frac{dx}{dt} = 2t + C_1$

$t = 0 \Rightarrow \frac{dx}{dt} = 0$ and $C_1 = 0$; $\frac{dx}{dt} = 2t \Rightarrow x(t) = t^2 + C_2$

$x(0) = \pi \Rightarrow C_2 = \pi$ and $x(t) = t^2 + \pi$

$y(t) = \cos(t^2 + \pi) = -\cos(t^2)$

$$\boxed{x(t) = t^2 + \pi;\quad y(t) = -\cos(t^2)}$$

b.) Find speed when position of particle is $(4, \cos 4)$

$x(t) = t^2 + \pi = 4 \Rightarrow t = \sqrt{4-\pi}$

Speed $= |\vec{v}| = \sqrt{(dx/dt)^2 + (dy/dt)^2}$ when $t = \sqrt{4-\pi}$

$\frac{dx}{dt} = 2t \Rightarrow \left.\frac{dx}{dt}\right|_{t=\sqrt{4-\pi}} = 2\sqrt{4-\pi}$

$\frac{dy}{dt} = 2t\sin(t^2) \Rightarrow \left.\frac{dy}{dt}\right|_{t=\sqrt{4-\pi}} = 2\sqrt{4-\pi}\sin(4-\pi) = -2\sqrt{4-\pi}\sin 4$

$|\vec{v}| = \sqrt{(2\sqrt{4-\pi})^2 + (-2\sqrt{4-\pi}\sin 4)^2}$

$$\boxed{|\vec{v}| = 2\sqrt{4-\pi}\sqrt{1+\sin^2 4}}$$

1992 - BC 3

3. At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the xy-plane is given by the parametric equations $x = e^t \sin t$ and $y = e^t \cos t$.

(a) Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

(b) Find the speed of the particle when $t = 1$.

(c) Find the distance traveled by the particle along the path from $t = 0$ to $t = 1$.

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a.) $\frac{dy}{dt} = e^t(-\sin t) + e^t \cos t = e^t(\cos t - \sin t)$

$\frac{dx}{dt} = e^t \cos t + e^t \sin t = e^t(\cos t + \sin t)$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$

$t = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = \frac{\cos\frac{\pi}{2} - \sin\frac{\pi}{2}}{\cos\frac{\pi}{2} + \sin\frac{\pi}{2}} = -1$

b.) $v = \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{e^{2t}(\cos t + \sin t)^2 + e^{2t}(\cos t - \sin t)^2}$

$= e^t \sqrt{\cos^2 t + 2\cos t \sin t + \sin^2 t + \cos^2 t - 2\cos t \sin t + \sin^2 t}$

$= e^t\sqrt{2}$

When $t=1$, speed is $e\sqrt{2}$

c.) $s = \int_{t=0}^{1} v\,dt = \int_0^1 e^t\sqrt{2}\,dt = e^t\sqrt{2}\Big]_0^1$

$= e\sqrt{2} - \sqrt{2} = (e-1)\sqrt{2}$

Distance traveled from $t=0$ to $t=1$ is $\sqrt{2}(e-1)$