Answers to Day 3 Homework

1. Since $x'(t) = 6t^2 - 42t + 72 = 6(t^2 - 7t + 12) = 6(t - 3)(t - 4) = 0$ when $t = 3$ and when $t = 4$, the answer is E.

2. Note that $a(t) = 3t^2 - 6t + 12$, so that $a'(t) = 6t - 6 = 0$ when $t = 1$. Computing the acceleration at the critical number and at the endpoints of the interval, we have $a(0) = 12$, $a(1) = 9$, and $a(3) = 21$. The maximum acceleration is 21, so the answer is D.

3. Note that $v(t) = 6t^2 - 48t + 90 = 6(t - 3)(t - 5)$ and $a(t) = 12t - 48 = 12(t - 4)$. The speed is increasing on $3 < t < 4$, where the velocity and the acceleration are both negative, and also for $t > 5$, where the velocity and the acceleration are both positive, so the answer is E.

4. Since $\dfrac{d}{dt}\left[3 + 4.1\cos(0.9t)\right]_{t=4} = 1.633$, the answer is C.

5. Since $v(2) = 2 + \displaystyle\int_1^2 \ln(1 + 2^t)\,dt = 3.346$, the answer is E.

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6. First find $\dfrac{d}{dt}\left[\sin t\right] = \cos t$ and $\dfrac{d}{dt}\left[e^{-2t}\right] = -2e^{-2t}$. Then graph $y_1 = \cos x$ and $y_2 = -2e^{-2x}$ in function mode with an x-window of $[0, 10]$ and a y-window of $[-1, 1]$. The two graphs intersect at three points, so the answer is D.

7. Distance $= \displaystyle\int_0^2 \left|v(t)\right| dt = \int_0^2 \left|3e^{(-\frac{1}{2})} \sin(2t)\right| dt = 2.261$, so the answer is D.

8. (a) $a(2) = v'(2) = -0.132$ or -0.133.

   (b) $v(2) = -0.436$. Since $a(2) < 0$, and $v(2) < 0$, the speed is increasing.

   (c) Note that $v(t) = 0$ when $\tan^{-1}(e^t) = 1$. The only critical number for y is $t = \ln(\tan 1) = 0.443$. Since $v(t) > 0$ for $0 \le t < \ln(\tan 1)$ and $v(t) < 0$ for $t > \ln(\tan 1)$, $y(t)$ has an absolute maximum at $t = 0.443$.

   (d) $y(2) = -1 + \displaystyle\int_0^2 v(t)\, dt = -1.360$ or -1.361.

   Since $v(2) < 0$ and $y(2) < 0$, the particle is moving away from the origin.

9. (a) Average acceleration of rocket A is

   $$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2.$$

   (b) Since the velocity is positive, $\displaystyle\int_{10}^{70} v(t)\, dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is

   $$20\left[v(20) + v(40) + v(60)\right] = 20(22 + 35 + 44) = 2020\,\text{ft}.$$

   (c) Let $v_B(t)$ be the velocity of rocket B at time t. Then

   $$v_B(t) = \int \frac{3}{\sqrt{t+1}}\, dt = 6\sqrt{t+1} + C.$$ Since $2 = v_B(0) = 6 + C$, then $C = -4$ and $v_B(t) = 6\sqrt{t+1} - 4$. Hence, $v_B(80) = 50 > 49 = v(80)$ and Rocket B is traveling faster at time $t = 80$ seconds.