Review Session Practice Problems

① Convert polar $(\sqrt{3}, \frac{\pi}{6})$ to rectangular

$x = \sqrt{3}\cos\frac{\pi}{6}$ $\quad$ $y = \sqrt{3}\sin\frac{\pi}{6}$

$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ $\quad$ $x = \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{2}$ $\quad$ $y = \sqrt{3}\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

② Find Area inside one petal of $r = 3\cos 3\theta$

$$A = \frac{1}{2}\int r^2 \, d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} (3\cos 3\theta)^2 \, d\theta = 2.356$$

③ Area Inside $r = 3\sin\theta$ but outside $r = 2 - \sin\theta$

$$A = 2\left[\frac{1}{2}\int_{\pi/6}^{\pi/2} (3\sin\theta)^2 - \frac{1}{2}\int_{\pi/6}^{\pi/2} (2-\sin\theta)^2 \, d\theta\right]$$

$$= 5.196$$

④ $\vec{v} = \cos 2t\,\vec{i} - 2\sin t\,\vec{j}$. When $t=0$ the particle is at $\langle 3, -2 \rangle$. Find the position vector

$$\vec{r} = \left(\frac{1}{2}\sin 2t + C_1\right)\vec{i} + \left(2\cos t + C_2\right)\vec{j}$$

$3 = \frac{1}{2}\sin 2(0) + C_1$ $\quad$ $-2 = 2\cos(0) + C_2$

$3 = 0 + C_1$ $\quad$ $-2 = 2 + C_2$

$3 = C_1$ $\quad$ $-4 = C_2$

5 given $x=\sqrt{t}$ $\quad y=\frac{1}{4}(t^2-4)$

b) find $\frac{d^2y}{dx^2}$

a) find equation of tangent line where $t=4$

$(2,3)$ $\quad \frac{dy}{dx} = \frac{\frac{1}{2}t}{\frac{1}{2}t^{-1/2}} = t^{3/2}\Big|_{t=4} \Rightarrow m=8$

c) what is the convavity at $t=4$?

$y-3=8(x-2)$

b) find $\frac{d^2y}{dx^2} \Rightarrow \frac{\frac{3}{2}t^{1/2}}{\frac{1}{2}t^{-1/2}} = 3t$

c) what is the concavity at $t=4$?

$3(4)=12>0$ C.U.

d) Find arclength of curve $1<t<5$

$$\int_1^5 \sqrt{\left(\tfrac{1}{2}t^{-\frac{1}{2}}\right)^2+\left(\tfrac{1}{2}t\right)^2}\,dt$$

$$\int_1^5 \sqrt{\frac{1}{4t}+\frac{t^2}{4}}\,dt = 6.1876$$

6) given $y=\sqrt{x}$. find arclength $0<x<3$

$$\int_0^3 \sqrt{1+\left(\tfrac{1}{2}x^{-1/2}\right)^2}\,dx$$

$$\int_0^3 \sqrt{1+\frac{1}{4x}}\,dx = 3.6114$$

7) Slope of tangent line to $r = 2\cos 3\theta$ where $\theta = \frac{\pi}{6}$

$$\frac{2\cos 3\theta \cos\theta + 3\cdot 2\sin 3\theta \sin\theta}{-2\cos 3\theta \sin\theta + 3\cdot 2\sin 3\theta \cos\theta} =$$

$$\frac{2\cdot 0\cdot \frac{\sqrt{3}}{2} + 6(1)\frac{1}{2}}{-2\cdot 0\cdot \frac{1}{2} + 6(1)\frac{\sqrt{3}}{2}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3}$$

8) Use Euler's method to approximate $y(0.2)$ of $y' = y$ with $y(0) = 1$ using two 2 steps

≈ 1.210

$x = 0 \quad y = 1$

$x = .1 \quad y = 1 + 1(.1) = 1.1$

$x = .2 \quad y = 1.1 + 1.1(.1) = 1.21$

7) $x = r\cos\theta \qquad y = r\sin\theta$

$x = 2\cos(3\cdot \pi/6)\cos \pi/6 \qquad y = 2\cos(3\cdot \pi/6)\sin \pi/6$

$x = 2\cos(\pi/2)\cos \pi/6 \qquad y = 2\cos(\pi/2)\sin \pi/6$

$x = 0 \qquad y = 0$

$y = \frac{\sqrt{3}}{3}x$

9) $e^{3y}\frac{dy}{dx} = x^2 \qquad y = \frac{1}{3}\ln(x^3 + D)$

$e^{3y}\,dy = x^2\,dx$

$\frac{1}{3}e^{3y} = \frac{x^3}{3} + C$

$e^{3y} = x^3 + 3C$

$3y = \ln(x^3 + 3c)$

(10)

$$\frac{dP}{dt} = .36P\left(1 - \frac{P}{26}\right)$$

$$\frac{dP}{dt} = .04P\left(1 - \frac{P}{50}\right)$$

$$= .04P\left(\frac{50}{50} - \frac{P}{50}\right)$$

$$= .04P\frac{1}{50}(50 - P)$$

$$= \frac{\overset{2}{\cancel{4}}}{100} \cdot \frac{1}{\underset{25}{\cancel{50}}}$$

$$\frac{dP}{dt} = \frac{2}{2500}P(50 - P)$$

entrance ticket

Name: ____________________

Solve the following systems of inequalities by graphing:

1. $3x + 4y < 28$
$x - 2y \leq 0$

$y = -x^2 + 6x - 5$

$\int_1^5 2\pi x(-x^2+6x-5)$

$y = Pe^{rt}$

$1000 = 500e^{r25}$

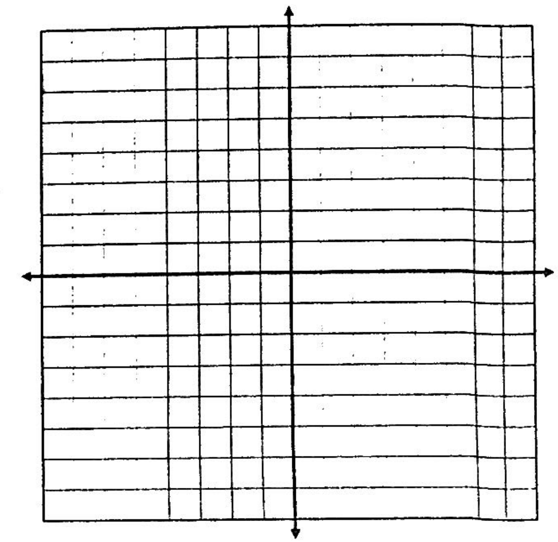
$2 = e^{25r}$

$\ln 2 = 25r$

$\frac{\ln 2}{25} = r$

$y = 500e^{r45}$

11



2. Which of the following points are solutions to the system above?

a. (1, 3) **b.** (3, -4) **c.** (0, -5) **d.** (-2, 0)