

| | | | | |
|------|---|---|---|---|
| x | 1 | 2 | 3 | 4 |
| f(x) | 4 | 2 | 3 | 1 |
| g(x) | 2 | 3 | 1 | 4 |

11. Selected values for continuous functions f(x) and g(x) are given in the table above. $\lim_{x \rightarrow 3} \frac{f(g(x))}{g(f(x))} = \frac{f(1)}{g(3)} = \frac{4}{1}$

- (A) $\frac{1}{4}$
 (B) $\frac{1}{3}$
 (C) $\frac{1}{2}$
 (D) 4

D

15. Which of the following statements is true?
 (A) $\lim_{x \rightarrow 3} \log_3 x = 2 = 1$
 (B) $\lim_{x \rightarrow 0^+} \log_3 x$ does not exist. $-\infty$
 (C) $\lim_{x \rightarrow -\infty} e^x$ does not exist. $= 0$
 (D) $\lim_{x \rightarrow 1} e^{x-1} = 0$ $e^0 = 1$

$\frac{\sin x}{e^x} = \frac{1}{1 + \frac{\cos x}{e^x}} = \frac{0}{1+0} = 0$

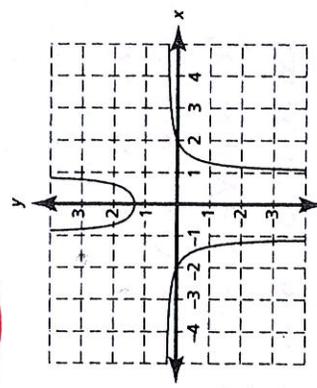
B

13. For what value of k is the function $f(x) = \begin{cases} 2x^2 + 5x - 3, & x \neq -3 \\ k, & x = -3 \end{cases}$ continuous at $x = -3$?

- (A) $-\frac{7}{6}$
 (B) 0
 (C) $\frac{5}{6}$
 (D) $\frac{7}{6}$

D

$\frac{(2(-3)-1)(-3+3)}{(-3-3)(-3+3)} = \frac{2(-3)-1}{-3-3} = \frac{-7}{-6} = \frac{7}{6}$



14. The function g(x) is shown in the graph above and is of the form $g(x) = \frac{x^2 + a}{bx^2 - 3}$. Which of the following could be the values of the constants a and b?

- (A) $a = -2, b = -1$
 (B) $a = -2, b = -3$
 (C) $a = -4, b = 3$
 (D) $a = -4, b = -3$

C

$b x^2 - 3 = 0$
 $b x^2 = 3$
 $x^2 = \frac{3}{b}$

$\frac{3}{b} = 1$

MULTIPLE-CHOICE QUESTIONS

Calculators may not be used for the following questions.

1. What does the limit statement $\lim_{x \rightarrow 1} \frac{\ln(x+1) - \ln 2}{x-1}$ represent?

- (A) 0
 (B) $\frac{d}{dx} [\ln(x+1)]$
 (C) $f'(1)$, if $f(x) = \ln(x+1)$
 (D) 1

C

2. Find the derivative of the function $y = \frac{4}{x^3} = 4x^{-3}$

- (A) $-\frac{12}{x^2}$
 (B) $\frac{12}{x^2}$
 (C) $\frac{12}{x^4}$
 (D) $-\frac{12}{x^4}$

D

3. Find $\frac{dy}{dx}$ if $3xy = 4x + y^2$.

- (A) $\frac{4-3y}{2y-3x}$
 (B) $\frac{3x-4}{2x}$
 (C) $\frac{3y-x}{2}$
 (D) $\frac{3y-4}{2y-3x}$

D

$3y + 3x \frac{dy}{dx} = 4 + 2y \frac{dy}{dx}$
 $3y + 3x \frac{dy}{dx} - 2y \frac{dy}{dx} = 4$
 $(3x-2y) \frac{dy}{dx} = 4-3y$
 $\frac{dy}{dx} = \frac{4-3y}{3x-2y}$

4. Find $\frac{dy}{dx}$ for $e^{xy} = y$.

- (A) $\frac{e^{xy}}{1-e^{xy}}$
 (B) $\frac{e^{xy}}{1+e^{xy}}$
 (C) $\frac{e^{xy}}{e^{xy}-1}$
 (D) e^{xy}

A

$e^{xy} (1+y') = y'$
 $e^{xy} + y' e^{xy} = y'$
 $e^{xy} = y' (1 - e^{xy})$

5. If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(n)}$ is the same as

- (A) y
 (B) $\frac{dy}{dx}$
 (C) $\frac{d^2y}{dx^2}$
 (D) $\frac{d^3y}{dx^3}$
- $y' = -\cos x$
 $y'' = \sin x$
 $y''' = \cos x$
 $y^{(4)} = -\sin x$

D

6. Find the second derivative of $f(x)$ if $f(x) = (2x+3)^4$.

- (A) $4(2x+3)^3$
- (B) $8(2x+3)^3$
- (C) $24(2x+3)^2$
- (D) $48(2x+3)^2$

$f'(x) = 4(2x+3)^3 \cdot 2 = 8(2x+3)^3$
 $f''(x) = 24(2x+3)^2 \cdot 2$

Calculators may be used for the following questions.

7. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.

- (A) $8\sin(3x)$
- (B) $8\sin(3x)\cos(3x)$
- (C) $12\sin(3x)\cos(3x)$
- (D) $24\sin(3x)\cos(3x)$

$y' = 8[\sin(3x)]' \cos(3x) = 3$

9. If $\ln y = (\ln x)^2 + 2$, find $\frac{dy}{dx}$ in terms of x and y .

- (A) $y \left[2\ln(x) + \frac{1}{x} \right]$
- (B) $y \left[\frac{2}{x} \ln(x) \right]$
- (C) $\left(\frac{2}{x} \right) \ln(x)$
- (D) $y \left[\frac{2\ln(x)+2}{x} \right]$

$\frac{1}{y} y' = 2(\ln x)' \cdot \frac{1}{x}$
 $y' = y \left[\frac{2}{x} \ln x \right]$

10. If $f(2) = -3$, $f'(2) = \frac{3}{4}$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

- (A) $y - 2 = \frac{-3}{4}(x+3)$
- (B) $y - 2 = \frac{-4}{3}(x+3)$
- (C) $y + 2 = \frac{4}{3}(x-3)$
- (D) $y - 2 = \frac{4}{3}(x+3)$

$g'(x) = \frac{1}{f'(g(x))}$
 $g'(-3) = \frac{1}{f'(2)} = \frac{1}{3/4} = 4/3$

11. For what positive value of x does the tangent line to the curve $y = \ln(1-x)$ intersect the y -axis at the point $(0, 2)$?

- (A) 0.382
- (B) 0.547
- (C) 0.667
- (D) 0.778

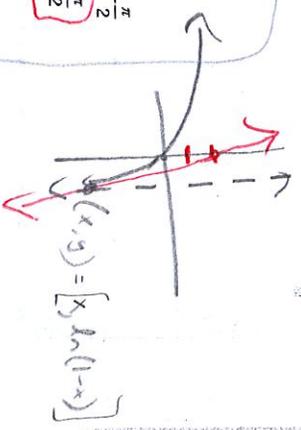
$y' = \frac{-1}{1-x}$
 $y - 2 = \frac{-1}{1-x}(x - 0)$

A calculator may not be used for the following questions.

12. For what values of a and c is the piecewise function

$f(x) = \begin{cases} ax^2 + \sin x, & x \leq \pi \\ 2x - c, & x > \pi \end{cases}$ differentiable?

- (A) $a = \frac{3c}{2}$ and $c = \frac{\pi}{2}$
- (B) $a = \frac{3}{2c}$ and $c = \frac{\pi}{2}$
- (C) $a = \frac{3}{2c}$ and $c = -\frac{\pi}{2}$
- (D) $a = \frac{3}{2c}$ and $c = \frac{\pi}{2}$



$a(\pi)^2 + \sin \pi = 2\pi - c$
 $\frac{3}{2}a(\pi)^2 + 0 = 2\pi - c$
 $\frac{3}{2}a\pi = 2\pi - c$
 $c = \frac{3}{2}a\pi$

$2a\pi + \cos \pi = 2$
 $2a\pi + (-1) = 2$
 $2a\pi = 3$
 $a = \frac{3}{2\pi}$

$\ln(1-x) - 2 = \frac{-1}{1-x}(x-0)$
 $\ln(1-x) - 2 + \frac{x}{1-x} = 0$
 $x = .77803632$

13. If $y = \tan^{-1}(x^2 + 3x)$, then $\frac{dy}{dx} =$

- (A) $\frac{1}{1+(x^2+3x)^2}$
- (B) $\frac{1}{x^2+3x+1}$
- (C) $\frac{2x+3}{1+(x^2+3x)^2}$
- (D) $\frac{2x+3}{x^2+3x}$

$\frac{1}{1+(x^2+3x)^2} \cdot 2x+3$

| x | f(x) | g(x) | f'(x) | g'(x) |
|---|------|------|-------|-------|
| 1 | 3 | 1 | -2 | 4 |
| 2 | 5 | 3 | 1 | -4 |
| 3 | 2 | 1 | -2 | 1 |
| 4 | 4 | -3 | 2 | -1 |

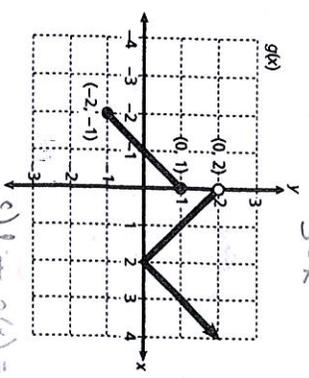
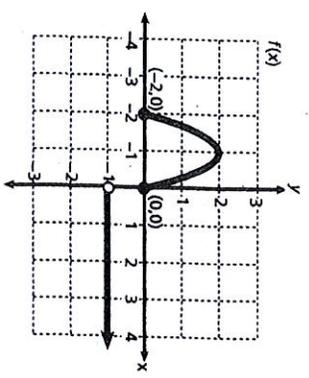
14. Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are given in the table above. If $p(x) = x \cdot f(x) - g(3x-2)$,

$p' = 1f(x) + x f'(x) - g'(3x-2) \cdot 3$
 $p'(2) = f(2) + 2f'(2) - g'(4) \cdot 3$
 $= 5 + 2 \cdot 1 - (-1) \cdot 3$
 $= 5 + 2 + 3$

FREE-RESPONSE QUESTION

A calculator may not be used for this question.

1. Use the graphs of $f(x)$ and $g(x)$ given below to answer the following questions:



- (a) Is $f(g(x))$ continuous at $x = 0$? Explain why or why not.
- (b) Is $g(f(x))$ continuous at $x = 0$? Explain why or why not.
- (c) What is $\lim_{x \rightarrow 0} f(g(x))$? Explain your reasoning.
- (d) If $h(x) = \begin{cases} f(x) + g(x), & -2 \leq x \leq 0 \\ k + g(x)f(x), & x > 0 \end{cases}$ continuous at $x = 0$?

(a) $\lim_{x \rightarrow 0} f(g(x)) = \infty$
 $\lim_{x \rightarrow 0} g(f(x)) = g(0) = -1$

(d) $f(0) = \begin{cases} 0, & x \leq 0 \\ -1, & x > 0 \end{cases}$

$f(1) = -1$, $f(g(0)) = -1$
 $f(2) = -1$
 $\lim_{x \rightarrow 0} f(g(x)) = -1$

\therefore continuous
 \therefore Not continuous

graphing calculators. (*Calculus for AP* 1st ed. pages 47–51, 65–68; EU 1.1; LO 1.1A; EK 1.1A2; MPAC 2)

FREE-RESPONSE QUESTION

| | Solution | Possible points |
|-----|---|---|
| (a) | $\lim_{x \rightarrow 0^-} f[g(x)] = \lim_{x \rightarrow 0^-} f[g(0^-)] = f(1) = -1.$ $\lim_{x \rightarrow 0^+} f[g(x)] = \lim_{x \rightarrow 0^+} f[g(0^+)] = f(2) = -1.$ $f[g(x)]$ is defined. The $\lim_{x \rightarrow 0} f[g(x)] = -1$ since the values from the right and the left are the same. Since $f(g(0))$ is equal to -1 , the limit exists and $\lim_{x \rightarrow 0} f[g(x)] = f[g(0)] = -1$. Yes, $f[g(x)]$ is continuous at $x = 0$. | $2: \begin{cases} 1: \text{ using left- and right-hand limits} \\ 1: \text{ answer "yes" with explanation} \end{cases}$ |
| (b) | $\lim_{x \rightarrow 0^-} g[f(x)] = \lim_{x \rightarrow 0^-} g[f(0^-)] = g(0) = 2.$ $\lim_{x \rightarrow 0^+} g[f(x)] = \lim_{x \rightarrow 0^+} g[f(0^+)] = g(-1) = 0.$ Therefore $\lim_{x \rightarrow 0} g[f(x)]$ does not exist since the limits from the left and right are not the same. No, $g[f(x)]$ is not continuous at $x = 0$. | $2: \begin{cases} 1: \text{ using left- and right-hand limits} \\ 1: \text{ answer "no" with explanation} \end{cases}$ |
| (c) | Since $\lim_{x \rightarrow \infty} g(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = -1$, then $\lim_{x \rightarrow \infty} f[g(x)] = -1$. | $2: \begin{cases} 1: \text{ value of each limit} \\ 1: \text{ answer} \end{cases}$ |
| (d) | $\lim_{x \rightarrow 0^-} f(x) + g(x) = \lim_{x \rightarrow 0^-} f(x) + \lim_{x \rightarrow 0^-} g(x) = 0 + 1 = 1$ and $h(0) = f(0) + g(0) = 0 + 1 = 1$. $\lim_{x \rightarrow 0^+} [k + f(x)g(x)] = k + \left(\lim_{x \rightarrow 0^+} f(x) \right) \left(\lim_{x \rightarrow 0^+} g(x) \right) =$ $k + (-1)(2) = k - 2.$ Therefore, $k - 2 = 1$ and $k = 3$. | $3: \begin{cases} 1: \text{ left-hand limit} \\ 1: \text{ right-hand limit} \\ 1: \text{ answer} \end{cases}$ |

(a), (b) (*Calculus for AP* 1st ed. pages 76–81; EU 1.2,1.1; LO 1.2A,1.1C; EK 1.2A1,1.1C1; MPAC 2,4)

(c) (*Calculus for AP* 1st ed. pages 23–30, 76–81; EU 1.1; LO 1.1C; EK 1.1C1; MPAC 2)

(d) (*Calculus for AP* 1st ed. pages 87–95; EU 1.1; LO 1.1C; EK 1.1C1,1.1C2; MPAC 2,3,4)