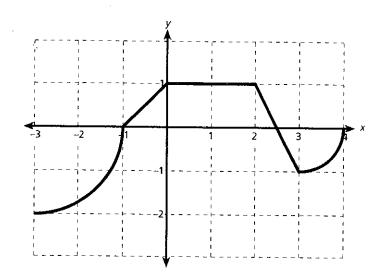
#### **142 ❖** CHAPTER 4

## **MULTIPLE-CHOICE QUESTIONS**

A calculator may not be used for the following questions.

- $1. \quad \int \sec^2 x 2 \, dx =$ 
  - (A)  $\tan x + C$
  - (B)  $\tan x 2x + C$
  - (C)  $\frac{\tan^3 x}{3} x + C$
  - (D)  $\frac{\sec^3 x}{3} 2x + C$
- 2. Find  $\int_{0}^{2} 3x^{2} f(x^{3}) dx$  if  $\int_{0}^{8} f(t) dt = k$ .
  - (A)  $k^3$
  - (B) 9k
  - (C) 3k
  - (D) k
- 3. If  $F(x) = \int_0^{x^2} \sqrt{t+5}$ , what is F'(2)
  - (A)  $\sqrt{7}$
  - (B) 3
  - (C) 12
  - (D) 18
- 4. What is the average value of  $f(x) = (\sin x) 4^{\cos x}$  for the closed interval  $0 \le x \le \frac{\pi}{2}$ ?
  - (A)  $\frac{3}{\ln 4}$
  - (B)  $\frac{6}{\pi \ln 4}$
  - (C)  $\frac{4}{\ln 4}$
  - (D)  $\frac{8}{\pi \ln 4}$
- 5. The graph of f(x) consists of line segments and quarter circles as shown in the graph to the right. What is the value
  - of  $\int_3^4 f(x) dx$ ?
  - (A)  $\frac{10-5\pi}{4}$
  - (B)  $\frac{10 + 5\pi}{4}$
  - (C)  $\frac{12+5\pi}{4}$
  - (D)  $\frac{12-5\pi}{4}$



A calculator may be used for the following questions.

- 6. Let R be the region between the function  $f(x) = x^3 + 6x^2 + 10x + 4$ , the x-axis, and the lines x = 0 and x = 4. Using the Trapezoidal Sum, compute the area when there are four equal subdivisions.
  - (A) 196
  - (B) 288
  - (C) 296
  - (D) 396
- 7. What is f(x) if  $f'(x) = \frac{x}{x^2 1}$  and f(2) = 0?
  - (A)  $f(x) = \frac{1}{2} \ln |x^2 1| \ln 3$
  - (B)  $f(x) = \frac{1}{2} \ln |x^2 1| \ln \sqrt{3}$
  - (C)  $f(x) = \frac{1}{2} \ln |x^2 1| + \ln \sqrt{3}$
  - (D)  $f(x) = \frac{1}{2} \ln |x| \frac{1}{2} x$
- 8. What is the average value of  $f(x) = 2 \ln x$ ? on the closed interval
  - $1 \le x \le 3$
  - (A) 2.592
  - (B) 2.000
  - (C) 1.296
  - (D) 1.952
- 9. Evaluate:  $\int_{1}^{0} \frac{x^2}{\sqrt[5]{2x^3 + 1}} dx$ .
  - (A)  $\frac{4}{15}$
  - (B)  $\frac{5}{12}$
  - (C) 0
  - (D) The function is not integrable on the interval  $-1 \le x \le 0$ .
- 10. If  $\int_1^3 f(x)dx = k$  and  $\int_1^7 f(x)dx = -4$ , what is the value of

$$\int_{7}^{3} x + f(x) dx ?$$

- (A) k 4
- (B) 16 k
- (C) -16 k
- (D) -16 + k

										ı
X	-4	-3	-2	-1	0	1	2	3	4	
f(x)	0.48	1.25	1.07	0.53	0.27	1.04	3.56	2.18	2	}

- 11. Selected values for the continuous function f(x) are given in the table above. Using three left-hand rectangles of equal width, an approximation for  $\int_{-3}^{3} f(x) dx$  is
- (A) 9.90
- (B) 7.72
- (C) 5.64
- (D) 4.90

12. Initially a water tank contains 100 ft<sup>3</sup>. Water begins to drain from

the tank at the rate of  $\frac{30e^{\frac{t}{3}}}{1+e^{\frac{t}{3}}}$  cubic feet per hour. How many cubic

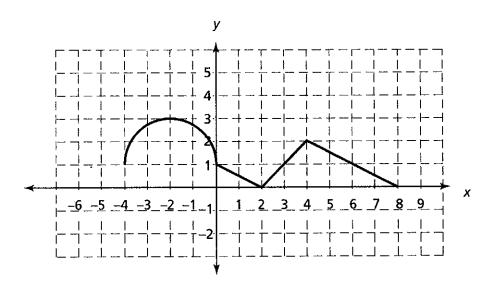
feet of water will remain in the tank after 3 hours?

- (A) 39.768
- (B) 44.190
- (C) 55.810
- (D) 60.232

A calculator may not be used on the following questions.

- 13. If R(x) is an even function and S(x) is an odd function where  $\int_{-a}^{0} R(x) dx = 5 \text{ and } \int_{0}^{a} S(x) dx = -3, \text{ find the value of } \int_{-a}^{a} \left[ 2(S(x)) R(x) + 3 \right] dx.$ 
  - (A) -10 + 3a
  - (B) -10 + 6a
  - (C) 10 + 6a
  - (D) -6 + 6a

- 14.  $\int \frac{9x}{\sqrt{1-81x^4}} \, dx =$ 
  - (A)  $(1-9x^2)^{\frac{1}{2}}+C$
  - (B)  $\frac{1}{2} (1 9x^2)^{\frac{3}{2}} + C$
  - (C)  $\sin^{-1}(9x^2) + C$
  - (D)  $\frac{1}{2}\sin^{-1}(9x^2) + C$



- 15. The graph of f(x) shown above consists of three line segments and one semicircle. Let  $g(x) = \int_{-2}^{x} f(t)dt$ . Which of the following statements must be false?
  - (A)  $g(4) = \int_{-2}^{4} f(t)dt = \pi + 5$
  - (B)  $g(-4) = \int_{-2}^{-4} f(t) dt = -\pi 2$
  - (C) g'(6) = 1
  - (D) g(x) has a relative maximum at x = 4.

### **FREE-RESPONSE QUESTION**

This question requires the use of a calculator.

- 1. The acceleration of a particle is given as  $a(t) = 3e^x x^4$  cm/sec<sup>2</sup> on the closed interval [0, 3] as illustrated on the graph shown.
  - (a) Find the velocity of the particle at any time t if v(0) = 6 cm/sec.
  - (b) Find the position of the particle at any time t if x(0) = -5 cm.
  - (c) At t = 6.5, is the speed of the particle increasing or decreasing? Explain your reasoning.
  - (d) On the closed interval [0, 3], what is the velocity of the particle when its

acceleration is at a maximum? Explain your reasoning.

# **MULTIPLE-CHOICE QUESTIONS**

A calculator may not be used for the following questions.

- $1. \quad \int 9x \cos(3x+1) dx =$ 
  - (A)  $9x\cos(3x+1) + \sin(3x+1) + C$
  - (B)  $3x\sin(3x+1)-\cos(3x+1)+C$
  - (C)  $3\sin(3x+1) \frac{1}{3}\cos(3x+1) + C$
  - (D)  $3x\sin(3x+1)+\cos(3x+1)+C$

X	g(x)	g'(x)	g"(x)	
0	2	-3	5	
1	3	8	2	

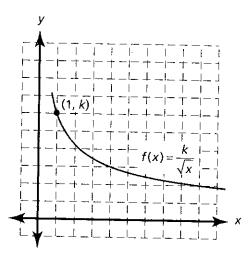
3. Let g be a twice-differentiable function with the selected values of g and its derivatives shown in the table above. What is the value of

$$\int_0^1 2xg''(x)dx?$$

- (A) 10
- (B) 12
- (C) 8
- (D) -4
- 4. Let  $f(x) = \frac{k}{\sqrt{x}}$ , where x > 0 and k is some finite positive constant, as pictured at the right.

Let  $L = \int_0^1 f(x) dx$  and  $M = \int_0^\infty f(x) dx$ . Which one of the following statements is true?

- (A) L < M
- (B) L = M
- (C) L > M
- (D) No conclusion can be made about the relative values of *L* and *M*.



- $6. \quad \int \frac{4\,dx}{(x-3)(x+1)} =$ 
  - (A)  $\ln |x-3| + \ln |x+1| + C$
  - (B)  $\ln \left| \frac{x-3}{x+1} \right| + C$
  - (C)  $4 \ln |(x-3)(x+1)| + C$
  - (D)  $4 \ln \left| \frac{x-3}{x+1} \right| + C$

8. Let f(x) be a differentiable function with the properties that f(1) = 5 and

$$\lim_{x\to\infty} f(x) = -8 \cdot \int_1^\infty f'(x) dx =$$

- (A) -13
- (B) -8
- (C) 5
- (D) ∞

10. 
$$\int \frac{2}{x^2 + 4x + 3} dx =$$

(A) 
$$\ln |x+1| + \ln |x+3| + C$$

(B) 
$$(\ln|x+1|)(\ln|x+3|)+C$$

(C) 
$$\ln \left| \frac{x+1}{x+3} \right| + C$$

(D) 
$$\frac{\ln|x+1|}{\ln|x+3|} + C$$

12. 
$$\int 2x^2e^{3x}dx =$$

(A) 
$$\frac{2}{3}x^2e^{3x} - \frac{4}{9}xe^{3x} + \frac{4}{27}e^{3x} + C$$

(B) 
$$2x^2e^{3x} + 4xe^{3x} + C$$

(C) 
$$6x^2e^{3x} - 36xe^{3x} + 108e^{3x} + C$$

(D) 
$$\frac{2}{3}x^2e^{3x} - \frac{4}{3}xe^{3x} + \frac{4}{3}e^{3x} + C$$

13. f(x) is a twice differentiable function on  $x \in [2,6]$  with selected values given in the table below.

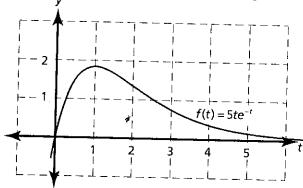
х	f(x)	f'(x)	f"(x)	
2	8	1	-1	
6	3	2	3 ,	

$$\int_2^6 x \cdot f''(x) dx =$$

- (A) 15
- (B) 20
- (C) 26
- (D) 34

# FREE-RESPONSE QUESTION

A calculator may not be used for this question.



Let  $F(x) = \int_0^x 5te^{-t} dt$  for  $t \ge 0$  and  $x \ge 0$ .

- (a) Find an expression for F'(x) in terms of x only.
- (b) Is the graph of F(x) concave up or concave down at x = 2? Explain your answer.
- (c) Find an expression for F(x), in terms of x only, that does not involve an integral.
- (d) Using your answer to c, find  $\lim_{x\to\infty} F(x)$ . Justify your answer.
- (e) Using your answer to d, explain what is meant by the expression  $\lim_{x\to\infty} F(x)$ .