Day 4 Review

FREE-RESPONSE QUESTION

A calculator may not be used on this question.

1. A differentiable function $f(x)$ is defined such that, for all values of x in its domain, $f(x) = 3 + \int_{8}^{x^3} f\left(\sqrt[3]{t}\right)\, dt$.
   (a) What is the domain of $f(x)$?
   (b) For what value(s) of x is $f(x) = 3$?
   (c) Show that $f'(x) = 3x^2 f(x)$.
   (d) Solve the differential equation in (c) to find $f(x)$ in terms of x only.

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used for the following questions.

An asterisk (*) indicates the question is for BC students.

1. The general solution to the differential equation $\frac{dy}{dx} = y^2 \sin x$ is
   (A) $y = \sqrt[3]{3\cos x + C}$
   (B) $y = -\cos x + C$
   (C) $y = \sqrt[3]{\sin x + C}$
   (D) $y = \frac{1}{\cos x + C}$

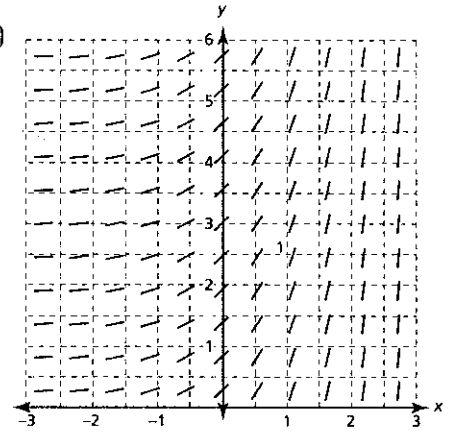
2. If $e^y \frac{dy}{dx} = 2x$ and $y(1) = 2$, then the particular solution $y(x)$ is
   (A) $y = \ln(x^2) + 2$.
   (B) $y = \ln(x^2 + e^2 - 1)$.
   (C) $y = 2e^{x^2-1}$.
   (D) $y = \ln(x^2 + e - 4)$.

Questions 3–5 refer to the following information:

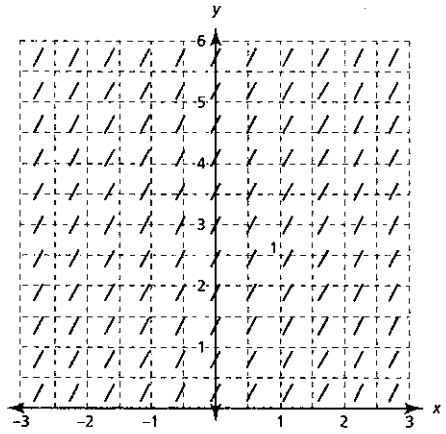
Consider the differential equation
$\frac{dy}{dx} = \frac{4x}{y}$, for $y \geq 1$ only, with initial value $y(0) = 1$.

3. Which of the following is the slope field for the general solution to the given differential equation?

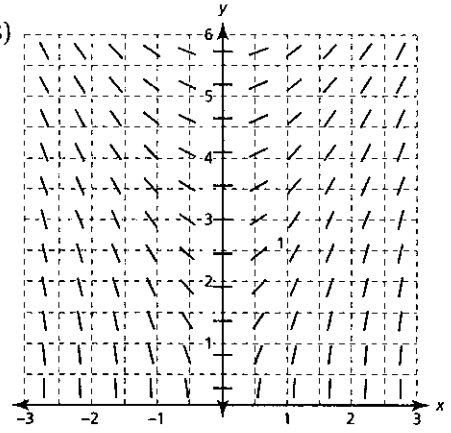
(A)



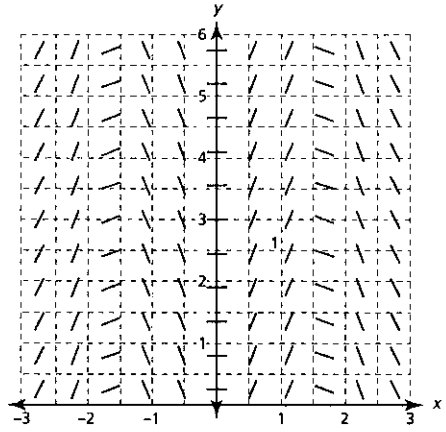
(C)



(B)



(D)



*4. Using Euler's Method with step size $\Delta x = 1/2$, what is the estimate for $y(1)$?
(A) 1
(B) 2
(C) $\sqrt{5}$
(D) 5/2

5. The particular solution $y(x)$ is
(A) $y = 2x$
(B) $y = \sqrt{4x^2 - 4}$
(C) $y = 2x^2 + 1$
(D) $y = \sqrt{4x^2 + 1}$

A calculator may be used for the following questions.

Questions 6–7 refer to the following information:

Water flows continuously from a large tank at a rate proportional to the amount of water remaining in the tank; that is, $\frac{dy}{dt} = ky$. There was initially 10,000 cubic feet of water in the tank, and at time $t = 4$ hours, 8000 cubic feet of water remained.

6. What is the value of k in the equation $\frac{dy}{dt} = ky$?

   (A) –0.050
   (B) –0.056
   (C) –0.169
   (D) –0.200

7. To the nearest cubic foot, how much water remained in the tank at time $t = 8$ hours?
   (A) 5778
   (B) 6000
   (C) 6400
   (D) 6458

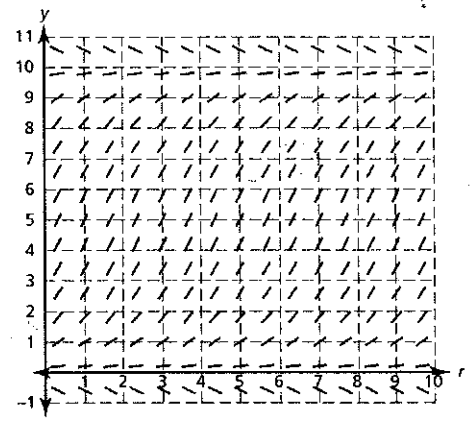
Questions 8–10 refer to the following information:

A population of rabbits in a certain habitat grows according to the differential equation $\frac{dy}{dt} = y\left(1 - \frac{1}{10}y\right)$ where t is measured in months ($t \geq 0$) and y is measured in hundreds of rabbits. There were initially 100 rabbits in this habitat; that is, $y(0) = 1$.

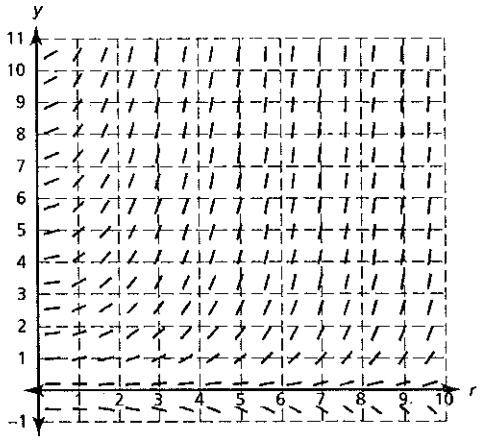
*8. What is the fastest growth rate, in rabbits per month, that this population exhibits?
   (A) 100
   (B) 200
   (C) 250
   (D) 500

*9. Which of the following slope fields represents an approximate general solution to the given differential equation?

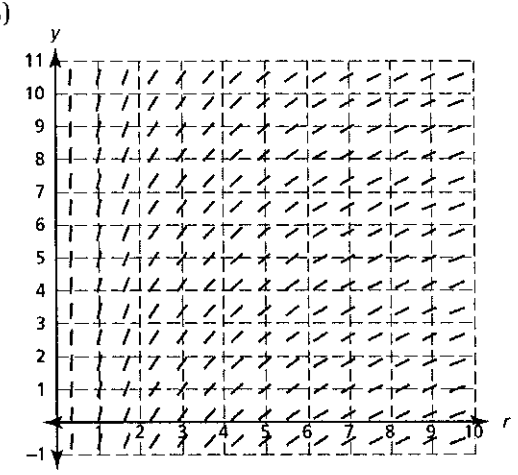
A)



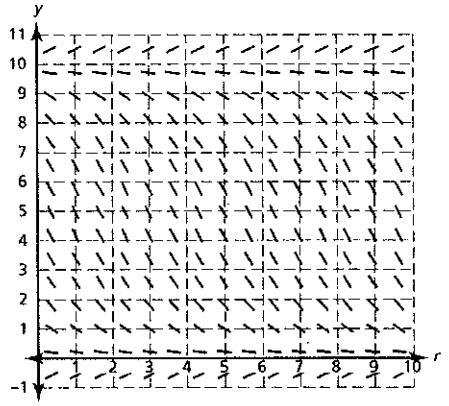
(C)



(B)



(D)



*10. Estimates of $y(t)$ can be produced using Euler's Method with step size $\Delta t = 1$. To the nearest rabbit, the estimate for $y(2)$ is

(A) 281
(B) 300
(C) 344
(D) 379

11. Water is being pumped continuously into a tank at a rate that is inversely proportional to the amount of water in the tank; that is, $\frac{dy}{dt} = \frac{k}{y}$, where y is the number of gallons of water in the tank after t minutes $(t \geq 0)$. Initially there were 5 gallons of water in the tank, and after 3 minutes there were 7 gallons. How many gallons of water were in the tank at time 18 minutes?

(A) $\sqrt{61}$
(B) $\sqrt{97}$
(C) 13
(D) 17

A calculator may not be used for the following questions.

12. $f'(x) = 12x^2 \sin(2x^3 - 16)$ and $f(2) = 5$ then $f(x) =$

(A) $-4x^3 \cos(2x^3 - 16) + 5$

(B) $2\cos(2x^3 - 16) + 3$

(C) $-2\cos(2x^3 - 16) + 7$

(D) $-2\cos(2x^2 - 16) + 5$

13. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2} y \cos(x)$, for which the solution is $y = f(x)$. Let $f(0) = 2$.

The particular solution is

(A) $f(x) = x + 2$

(B) $f(x) = 2e^{-\frac{1}{2}\sin(x)}$

(C) $f(x) = e^{\frac{1}{2}\sin(x)}$

(D) $f(x) = 2e^{\frac{1}{2}\sin(x)}$

Questions 14 and 15 refer to the following information:

Consider the differential equation $\frac{dy}{dx} = x + 2y$, for which the solution is $g(x)$.

14. Which of the following statements is true about the particular solution that contains $(0, -1)$?

(A) $g(x)$ is increasing and concave up.
(B) $g(x)$ is increasing and concave down.
(C) $g(x)$ is decreasing and concave up.
(D) $g(x)$ is decreasing and concave down.

*15. Let $y(x)$ be the particular solution that contains $(0,1)$. Using Euler's Method with step size $\Delta x = \frac{1}{2}$, what is the estimate for $y\left(\frac{1}{2}\right)$?

(A) $-\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{3}{2}$

(D) 2