 Day 5 Applications of Integration

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used for the following questions.

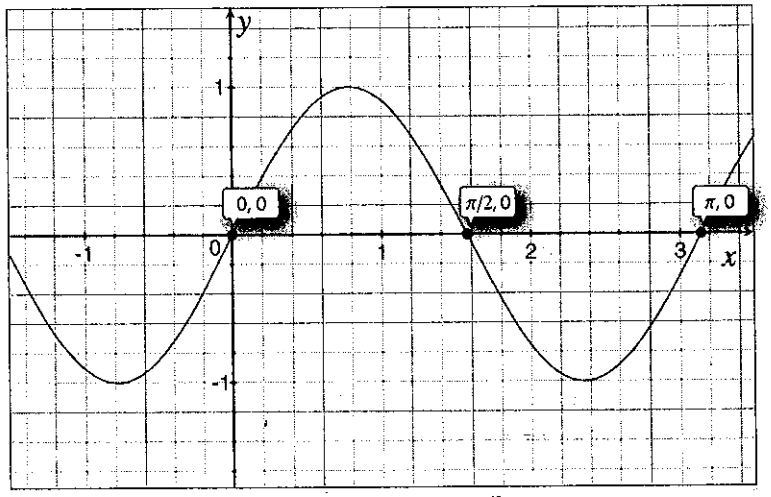
For Questions 1 and 2, region R is bounded by $f(y) = y^2 - 3$ and $g(y) = 3y + 1$.

1. Which of the following expressions gives the area of region R?
   (A) $\int_{-2}^{13} (3y+1) - (y^2 - 3)\, dy$
   (B) $\int_{-2}^{13} (y^2 - 3) - (3y+1)\, dy$
   (C) $\int_{-1}^{4} (3y+1) - (y^2 - 3)\, dy$
   (D) $\int_{-1}^{4} (y^2 - 3) - (3y+1)\, dy$

2. Which of the following expressions gives the volume when region R is rotated about the line $x = 4$?
   (A) $\pi \int_{-2}^{13} (7 - y^2)^2 - (3 - 3y)^2\, dy$
   (B) $\pi \int_{-1}^{4} (1 - y^2)^2 - (5 - 3y)^2\, dy$
   (C) $\pi \int_{-1}^{4} (7 - y^2)^2 - (3 - 3y)^2\, dy$
   (D) $\pi \int_{-2}^{13} (3y+1)^2 - (y^2 - 3)^2\, dy$

For problems 3 and 4, region Q is bounded by $y = \sin 2x$, $y = 0$, $x = \frac{\pi}{2}$ and $x = \pi$.

3. What is the area of region?
   (A) 0
   (B) $\frac{1}{2}$
   (C) $\frac{\pi}{2}$
   (D) 1



4. Which of the following expressions gives the volume of a solid whose base in the xy-plane is region Q and whose cross sections, perpendicular to the x-axis, are squares with a side in the xy-plane?

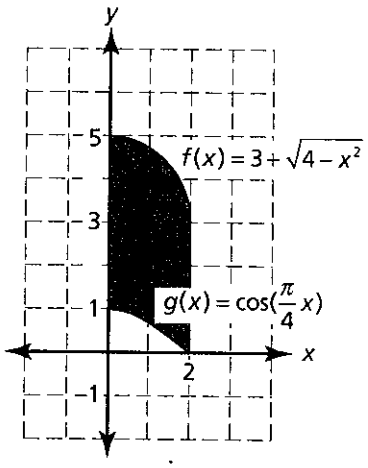
(A) $\pi \int_{\frac{\pi}{2}}^{\pi} \left(1 - \cos^2 2x\right) dx$

(B) $\int_{\frac{\pi}{2}}^{\pi} \left(-\sin 2x\right)^2 dx$

(C) $\pi \int_{\frac{\pi}{2}}^{\pi} \left(1 - \cos\ 2x\right) dx$

(D) $\pi \int_{\frac{\pi}{2}}^{\pi} \left(-\sin 2x\right)^2 dx$

For Question 5, region W is bounded by $f(x) = 3 + \sqrt{4 - x^2}$, $g(x) = \cos\left(\frac{\pi}{4}x\right)$, $x = 0$, and $x = 2$.



5. What is the area of the region W?

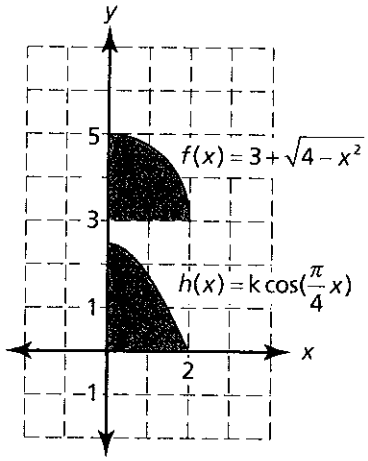
(A) 6.000

(B) $6 + \pi - \frac{4}{\pi}$

(C) $6 + \pi + \frac{4}{\pi}$

(D) $6 + 2\pi - \frac{4}{\pi}$

6. In the closed interval $0 \le x \le 2$, if $h(x) = k\cos\left(\frac{\pi}{4}x\right)$, for what value of k does the region bounded by $f(x)$ and the line $y = 3$ have the same area as that bounded by $h(x)$ and the x-axis?



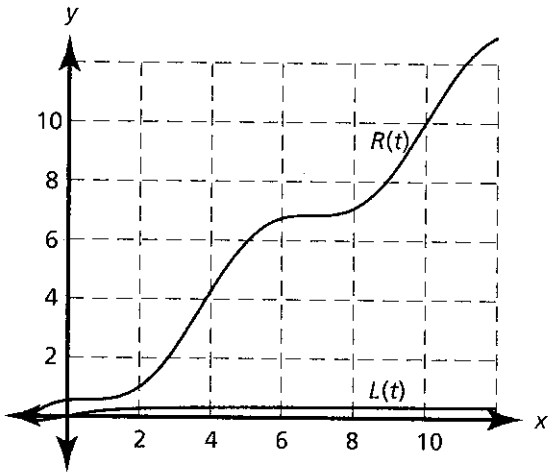
(A) $\frac{1}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi^2}{4}$

(D) $\frac{\pi^2}{2}$

A calculator may be used on the following questions.



7. Air is being pumped into a spherical balloon at the rate of $R(t) = t + \cos(t+1)\text{cm}^3/\text{min}$, but there is a small hole in the balloon and at the same time air is leaking out at the rate of $L(t) = 0.25\tan^{-1}(t)\text{cm}^3/\text{min}$. Which of the following is true for $0 \le t \le 10$, where t is measured in minutes?

(A) $\int_0^{10} R(t) - L(t)dt$ represents the volume of the balloon after the first 10 minutes.

(B) $\frac{1}{10}\int_0^{10} R(t) - L(t)dt$ represents the increase in volume each minute during the first 10 minutes.

(C) $\int_0^{10} R(t) - L(t)dt$ represents the increase in volume during the first 10 minutes.

(D) $R(0) - L(0) = 1$

8. The revenue and expenditures for a small company are analyzed and predicted for the next 5 years. The current annual revenue is \$125,000 and growing at an annual rate of 30%, while the expenditures are modeled with a sinusoidal function. If

$$R(t) = 125{,}000(1.3)^t \text{ and } E(t) = 25{,}000e^{\frac{t}{2}}\left(\sin\left(\frac{t}{2}\right) + \cos\left(\frac{t}{3}\right)\right),$$

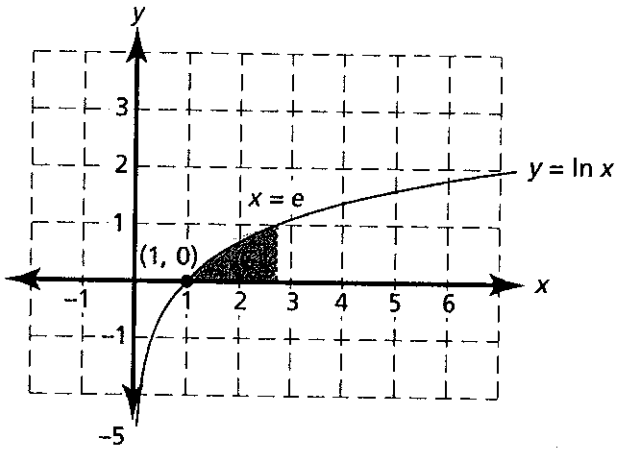
to the nearest dollar, what is the average annual profit for the company when the expenditures reach a maximum value on the interval $0 \le t \le 5$?

(A) \$434,691
(B) \$340,827
(C) \$127,282
(D) \$106,613

9. Region G is bounded by the curve $y = \ln x$, $x = e$, and the x-axis. Order from smallest to largest the volumes determined when G is rotated about the axes:

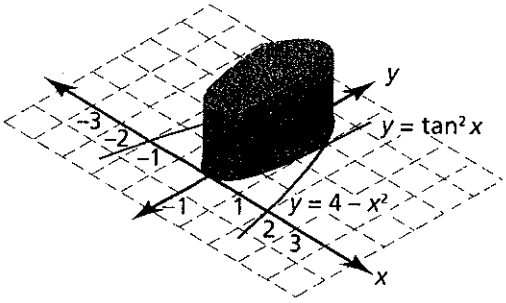
I. $y = 0$ II. $y = 1$ III. $y = e$

(A) III < I < II
(B) II < I < III
(C) I < II < III
(D) I < III < II

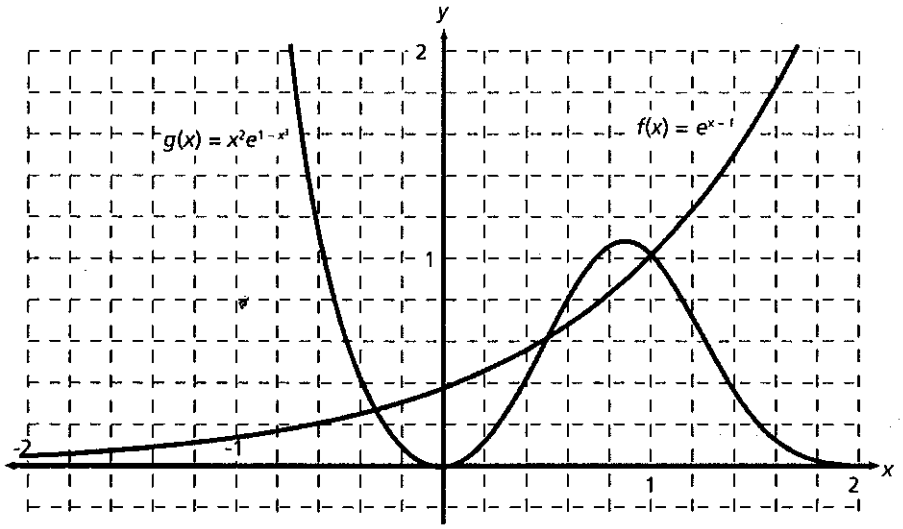


10. The base of a solid is bounded by $y = \tan^2 x$ and $y = 4 - x^2$ in the xy-plane, as shown in the figure below. Each cross section perpendicular to the x-axis is a rectangle with one side in the x-y plane and whose height is 2. What is the volume of the solid?

(A) 3.121
(B) 4.454
(C) 6.243
(D) 12.486



A calculator is required for the following questions.



11. Find the combined area of the two regions bounded by the curves $f(x) = e^{x-1}$ and $g(x) = x^2e^{1-x^3}$ as shown in the figure above.
    (A) 0.073
    (B) 0.203
    (C) 0.276
    (D) 0.931

Questions 12–14 refer to the following information:

The velocity of a particle moving along a line is given by $v(t) = e^t \sin\left(e^t\right)$ for the interval $0 \le t \le \frac{\pi}{2}$.

12. How far to the right of the starting point will the particle be at $t = \frac{\pi}{2}$?
    (A) 0.442
    (B) 0.540
    (C) 1.145
    (D) 1.540

13. What is the distance that the particle has traveled on this time interval?
    (A) 0.442
    (B) 1.145
    (C) 1.540
    (D) 2.638

14. If the position of the particle at time $t = 0$ is -1, then what is the position of the particle when it is farthest to the right on this time interval?
    (A) 0.442
    (B) 0.540
    (C) 1.145
    (D) 1.540

15. Which of the following expressions represents the volume of the solid generated when the region bounded by $y = x^2 - 2x + 3$ and $y = 1 + 3x - x^2$ is rotated about the line $y = -2$?

(A) $V = \pi \int_{\frac{1}{2}}^{2} \left[(1 + 3x - x^2) - (x^2 - 2x + 3) \right] dx$

(B) $V = \pi \int_{\frac{1}{2}}^{2} \left[(-1 + 3x - x^2)^2 - (x^2 - 2x + 1)^2 \right] dx$

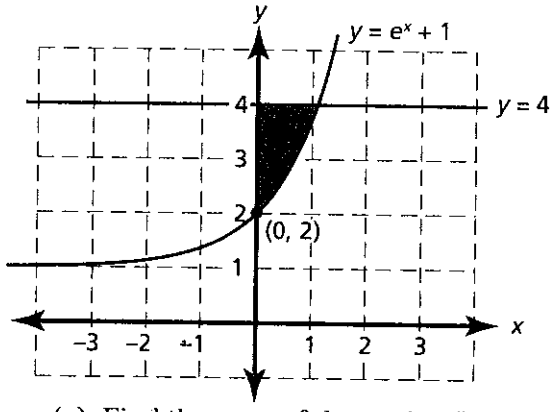
(C) $V = \pi \int_{\frac{1}{2}}^{2} \left[(3 + 3x - x^2)^2 - (x^2 - 2x + 5)^2 \right] dx$

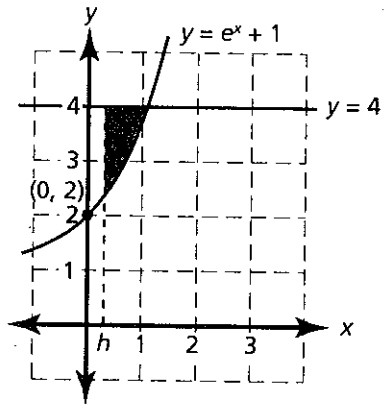
(D) $V = \pi \int_{\frac{1}{2}}^{2} \left[(1 + 3x - x^2) - (x^2 - 2x + 3) \right]^2 dx$

FREE-RESPONSE QUESTION

A calculator may be used for this free-response question.

Let f be the function given by $f(x) = e^x + 1$ as shown in the sketch below, where the region R is bounded by the graph of $f(x)$, the y-axis, and the horizontal line $y = 4$.





(a) Find the area of the region R.

(b) A vertical line $x = h$, where $h > 0$ is chosen so that the area of the region bounded by $f(x)$, the y-axis, the horizontal line $y = 4$, and the line $x = h$ is half the area of region R. What is the value of h?

(c) Find the volume of the solid formed when region R is rotated about the line $y = 4$.

(d) A horizontal line $y = k$, where k is greater than 4, is chosen so that the volume of the solid formed when region R is rotated about the line $y = k$ is twice the volume of the solid found in part (c). Set up, but do *not* evaluate, an integral expression in terms of a single independent variable which represents the volume of this solid.

2. Which of the following integral expressions represents the area of the region bounded by the graphs of $y = \frac{9}{x^2 + x - 20}$, $x = -3$, $x = 2$, and the x-axis?

(A) $\left| \int_{-3}^{2} \left(\frac{1}{x-4} + \frac{1}{x+5} \right) dx \right|$

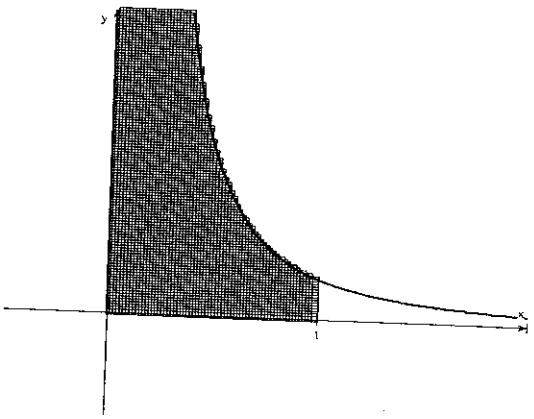
(B) $\left| \int_{-3}^{2} \left(\frac{1}{x-4} - \frac{1}{x+5} \right) dx \right|$

(C) $\left| \int_{-3}^{2} \left(\frac{5}{x-4} + \frac{4}{x+5} \right) dx \right|$

(D) $\left| \int_{-3}^{2} \left(\frac{1}{x+4} - \frac{1}{x-5} \right) dx \right|$

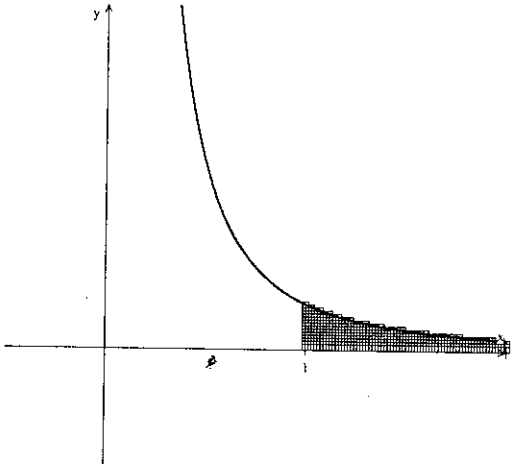
5. What is the area of the region in the first quadrant between the x-axis, y-axis, $y = \frac{1}{x^2}$, and $x = 1$.

(A) 1
(B) 2
(C) 0
(D) ∞



7. What is the area of the region in the first quadrant between the x-axis , the graph $y = \frac{1}{x^2}$, and $x = 1$?

(A) 1
(B) 2
(C) 0
(D) ∞



9. The area of the region bounded by the graphs of $y = xe^x$, $x = 0$, $x = \ln 2$, and the x-axis is

(A) $2\ln(2) - 1$

(B) $2\ln(2)$

(C) $2\ln(2) + 1$

(D) $[\ln(2)]^2 - 1$

11. The area of the Quadrant I region under the function $f(x) = \dfrac{e^x}{1 + e^{2x}}$ is

(A) $\dfrac{\pi}{4}$

(B) 1

(C) $\dfrac{\pi}{2}$

(D) divergent

14. Let R be the region bounded by the graphs of the function $f(x) = \dfrac{1}{\sqrt{x^2 - 1}}$, the x-axis, and the lines $x = p$ and $x = q$, where $1 < p < q$. An expression for the volume of the solid generated by revolving R about the x-axis is

(A) $\displaystyle\int_p^q \frac{1}{x^2 - 1}\,dx$

(B) $\displaystyle\frac{\pi}{2}\int_p^q \left[\frac{1}{x-1} + \frac{1}{x+1}\right]dx$

(C) $\displaystyle\pi\int_p^q \left[\frac{1}{x-1} - \frac{1}{x+1}\right]dx$

(D) $\displaystyle\frac{\pi}{2}\int_p^q \left[\frac{1}{x-1} - \frac{1}{x+1}\right]dx$

15. The base of a certain solid is the region bounded by the coordinate axes, the line $x = 16$, and the graph of $y = x^{-\frac{1}{4}}$. For this solid, each cross section perpendicular to the x-axis is a square whose side is across the base. The volume of this solid is

(A) 8

(B) $\dfrac{32}{3}$

(C) 8π

(D) divergent