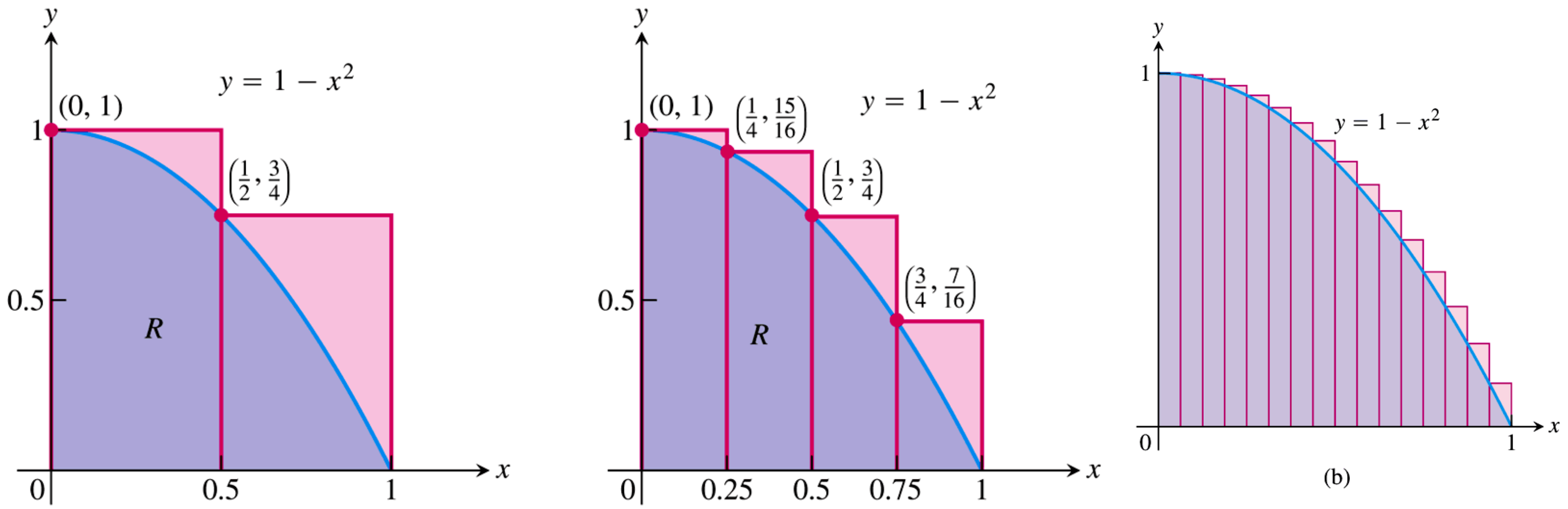
Section 5.2 The Definite Integral Using Geometry

- We have approximated the area under a curve using rectangles and trapezoids, but we would prefer to be exact.

As the number of rectangles increased, the approximation of the area under the curve approaches the exact area.



To be exact we need the number of rectangles to approach infinity

- If ***f(x)*** is a **<u>nonnegative</u>, continuous function** on the closed interval [a, b], then **the area of the region under the graph** of ***f(x)*** is given by

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$$

- where $\Delta x = \dfrac{b-a}{n}$
- n = number of subintervals on the interval (a,b)
- x_i = point in subinterval $x_i = a + i\Delta x$, based on right hand rectangles
- Δx is the width of our rectangles
- $f(x_i)$ is the height of our rectangle at the point x_i

This limit of the Riemann sum is also known as the definite integral of *f(x)* on [a, b]

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

This is read “the integral from a to b of *f* of *x* with respect to *x.”*

Sample Exam Questions

$dx = \Delta x = \frac{b-a}{n}$ $x_k = a + k\Delta x$

Which of the following limits is equal to $\int_2^5 x^2 dx$?

(A) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{k}{n}\right)^2 \frac{1}{n}$

(B) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{k}{n}\right)^2 \frac{3}{n}$

(C) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{3k}{n}\right)^2 \frac{1}{n}$

(D) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{3k}{n}\right)^2 \frac{3}{\text{n}}$

Which of the following integral expressions is equal to $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(\sqrt{1 + \frac{3k}{n}} \cdot \frac{1}{n} \right)$?

(A) $\int_0^1 \sqrt{1 + 3x}\, dx$

(B) $\int_0^3 \sqrt{1 + x}\, dx$

(C) $\int_1^4 \sqrt{x}\, dx$

(D) $\frac{1}{3} \int_0^3 \sqrt{x}\, dx$

Notation for the definite integral

upper limit of integration

Integration Symbol (integral)

$$\int_a^b f(x)\,dx$$

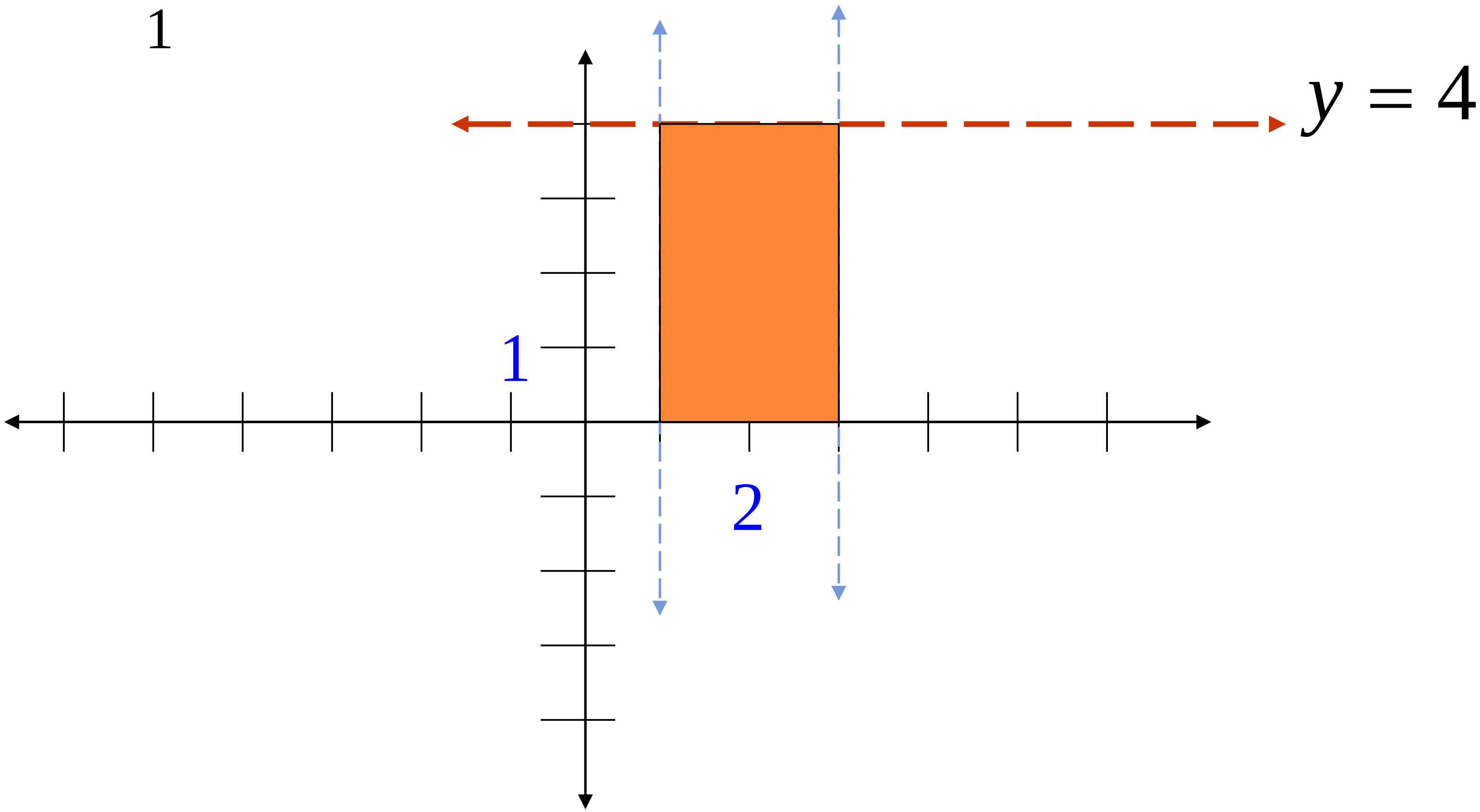
integrand

variable of integration

lower limit of integration

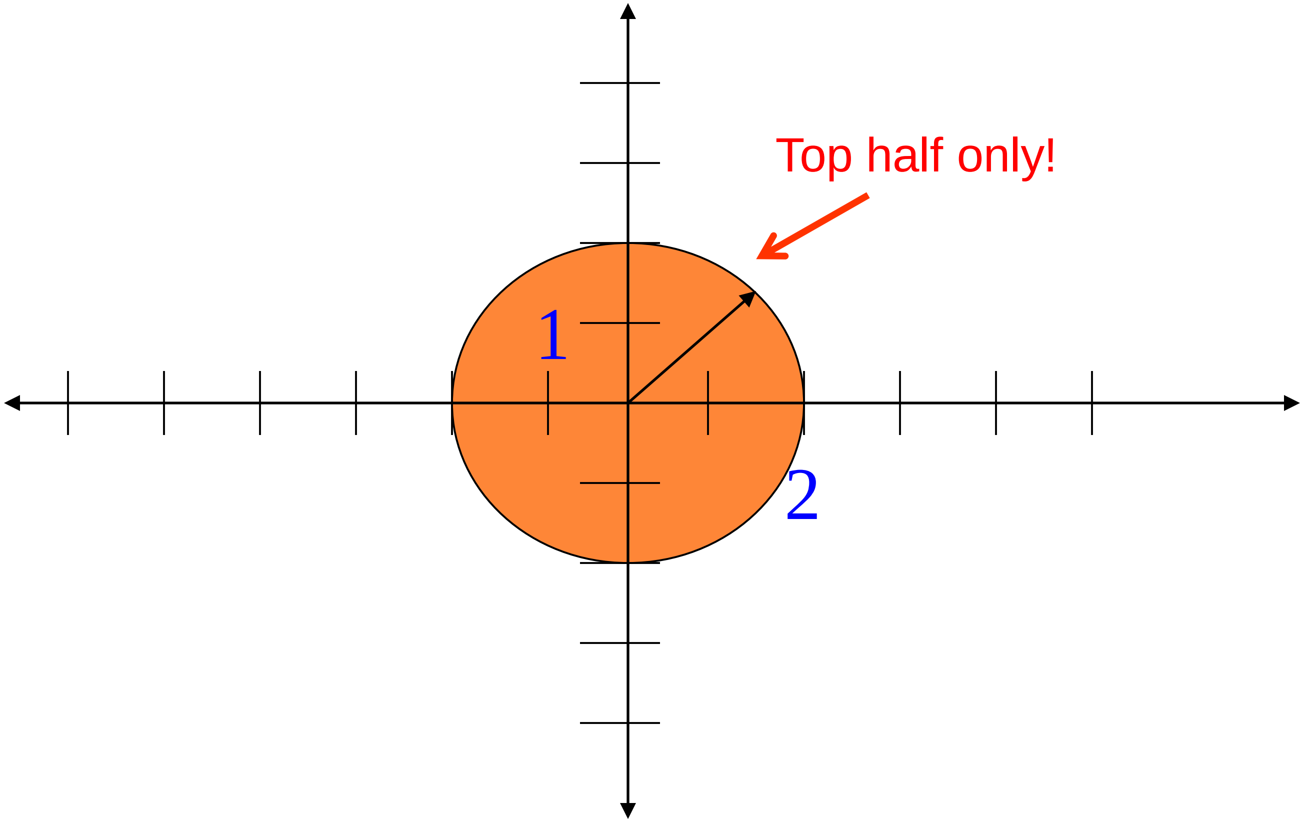
Evaluate the following definite integrals using geometric area formulas.

$$\int_{1}^{3} 4\,dx$$

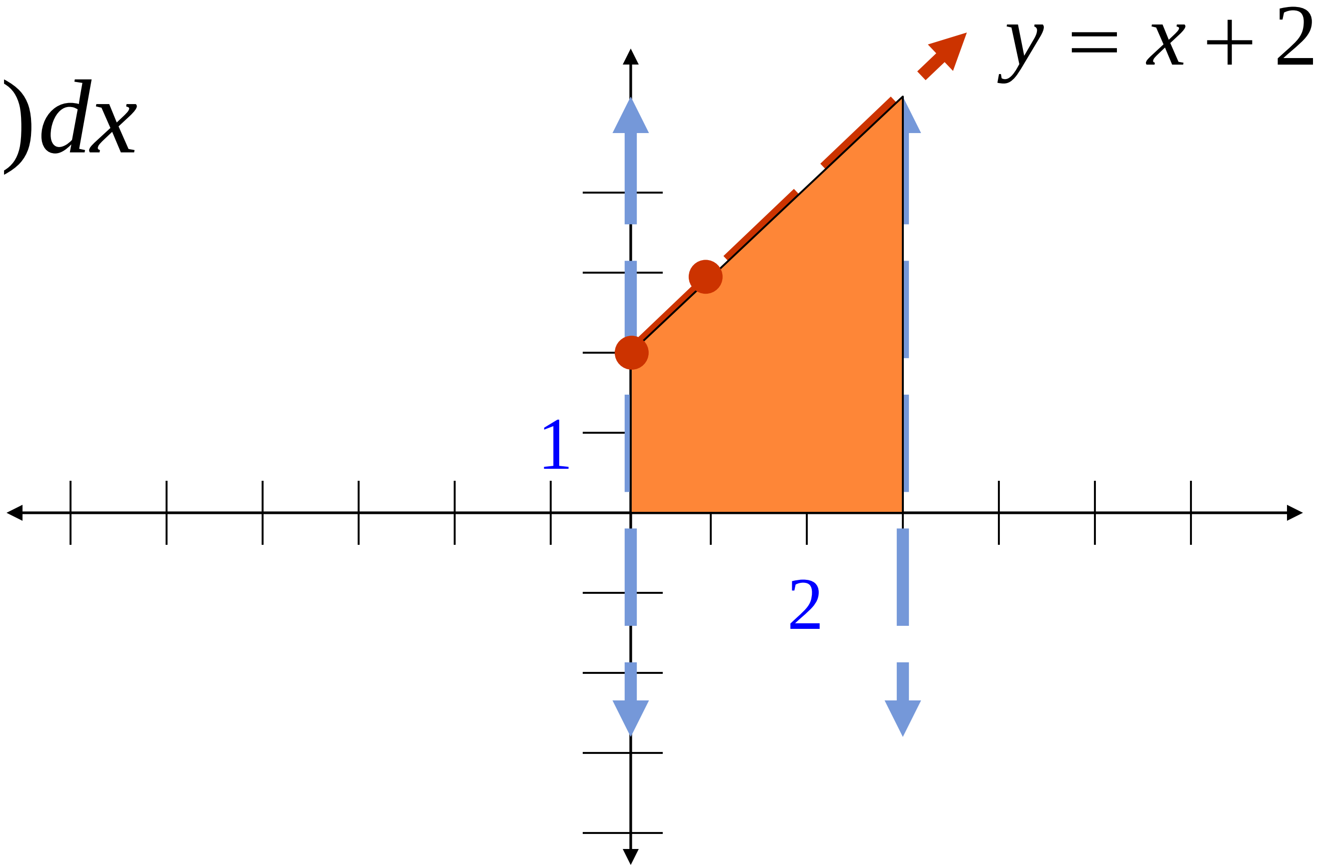


$$\int_{-2}^{2} \sqrt{4 - x^2}\, dx$$

$$y = \sqrt{4 - x^2} \Rightarrow x^2 + y^2 = 4$$

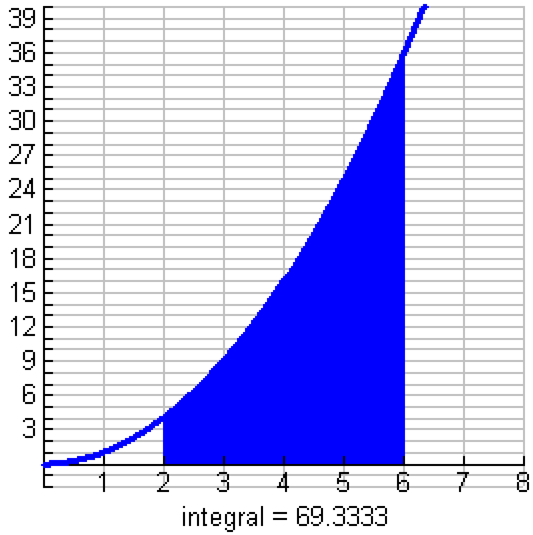


$$\int_0^3 (x+2)\,dx$$



We cannot use a formula to find the area under a curve, so we will use the calculator.

fnInt is option 9 under MATH key



fnInt(function, variable of integration, lower bound, upper bound)

$$\int_2^6 x^2\,dx = fnInt\left(x^2, x, 2, 6\right) = 69\frac{1}{3}$$

Evaluate the following integrals using your calculator

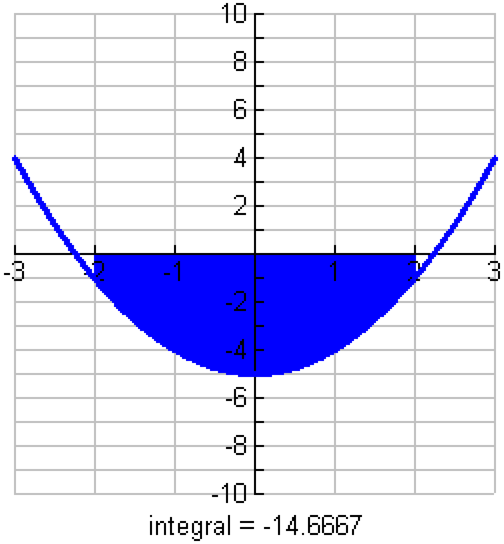
$$\int_{0}^{5} 3x^2 + 2x \, dx = 150$$

$$\int_{-2}^{8} 4x^2 + 3x \, dx = 783\frac{1}{3}$$

$$\int_{-4}^{4} 6x^2 \, dx = 256$$

Evaluate the following integral using your calculator

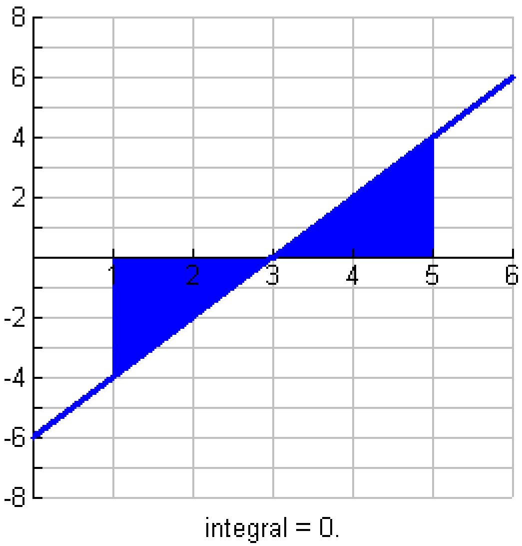
$$\int_{-2}^{2} x^2 - 5\,dx = -14\frac{2}{3}$$



Integrals treat areas <u>beneath</u> the x-axis as <u>*negative*</u>

Evaluate the following integral using your calculator

$$\int_{1}^{5} 2x-6\,dx = 0$$



Areas above and below the x-axis can cancel each other out.

For $f(x)$ *shown, Find*

$$\int_{-2}^{2} f(x)dx$$

$$\int_{-6}^{-2} f(x)dx$$

$$\int_{-6}^{6} f(x)dx$$