Section 10.2

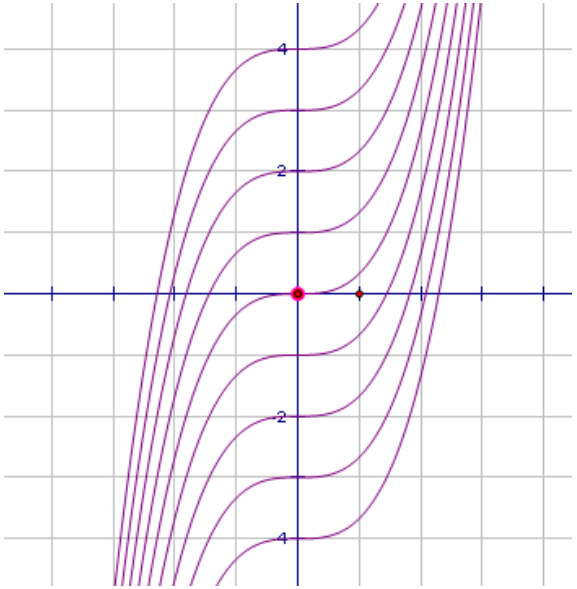
Differential Equations and Slope Fields

- A differential equation is an equation involving a derivative. The solution is the function y.

- Solve $\frac{dy}{dx} = x^2$, y(1) = 4

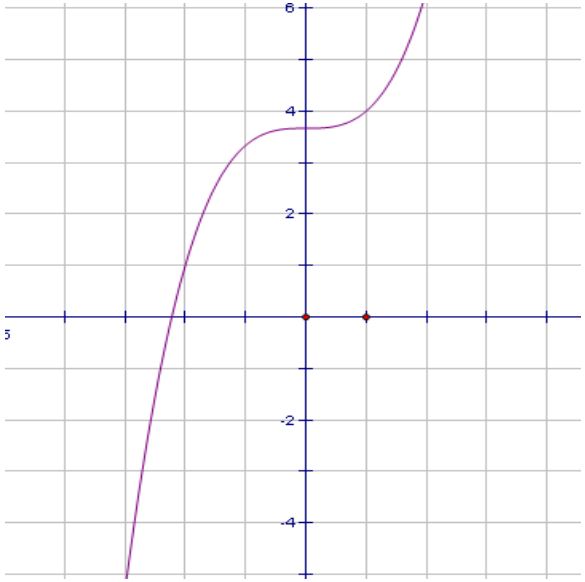
General solution

$$y = \frac{1}{3}x^3 + C$$



Particular solution

$$y = \frac{1}{3}x^3 + 3.667$$

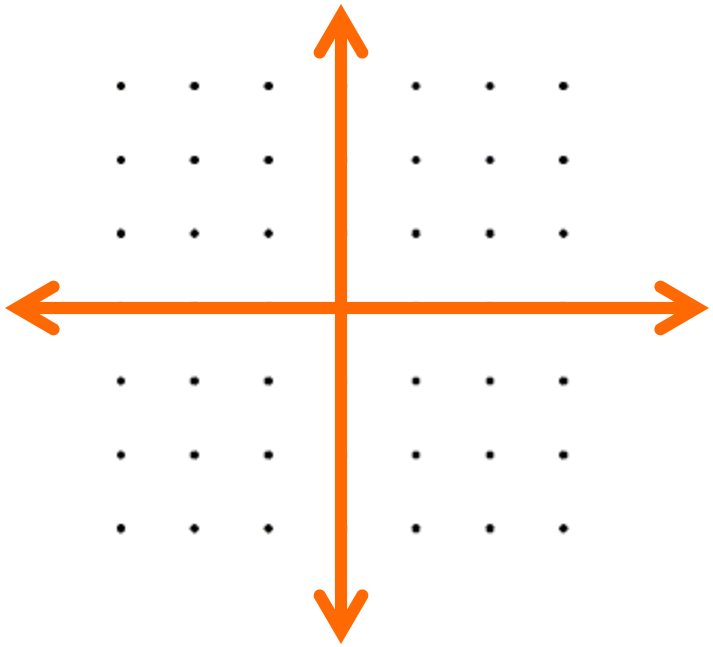


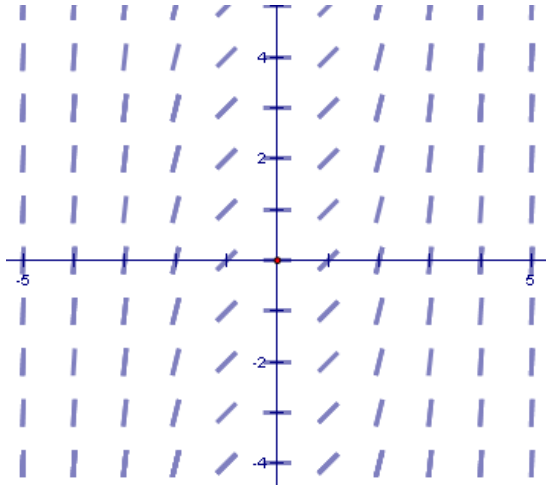
- What if the function is too complicated to find the antiderivative?

- A **slope field** (aka "direction field") is a tool to estimate the solution graph without actually integrating the differential equation.

Draw a slope field for $\frac{dy}{dx} = x^2$

x	dy/dx



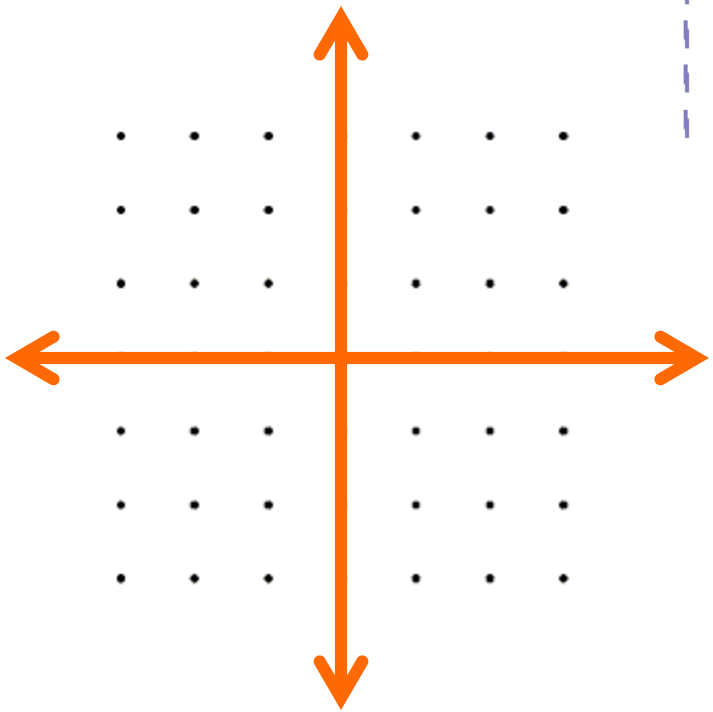


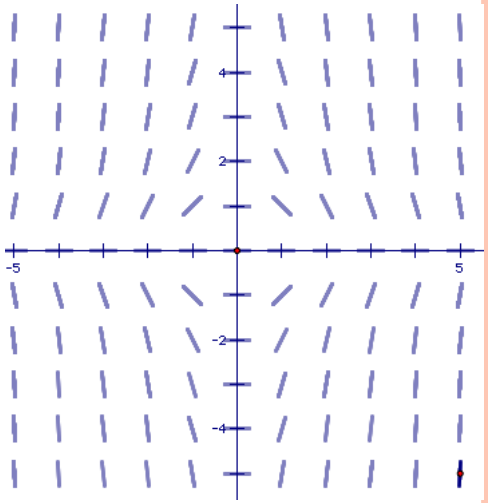
Computer generated slope field

Draw the particular solution through y(0) = 1

Solve $\frac{dy}{dx} = -xy$

x	y	dy/dx





Your Drawings should clearly show

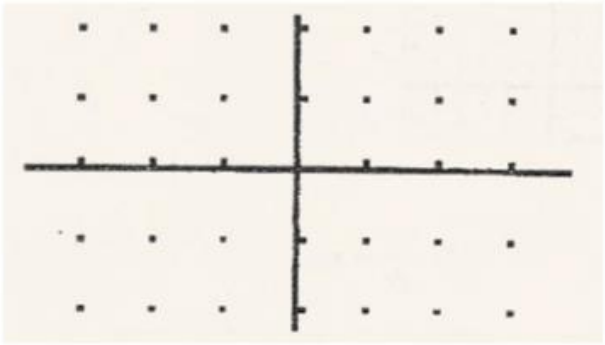
- Positive vs. negative slopes
- Slopes of 1 and -1
- Whether slopes are increasing or decreasing
- Zero and undefined slopes
- Do NOT plot at points where dy/dx is indeterminate (0/0)
- Symmetry

Specific solutions should

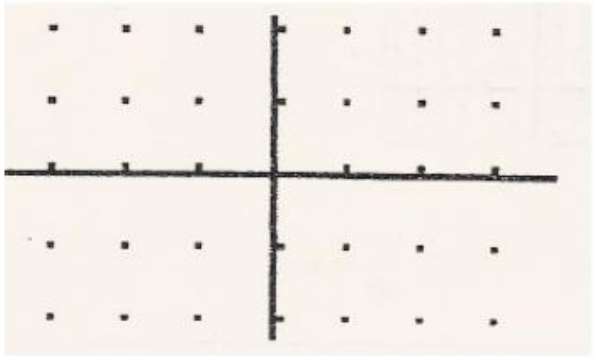
- Show point plotted at the given initial condition
- Flow through the slope field from edge to edge
- Be functions (i.e. pass the Vertical Line Test)
- Never, never, never cross a slope line!
- Never cross a gap, DNE, or vertical tangent.

You try:

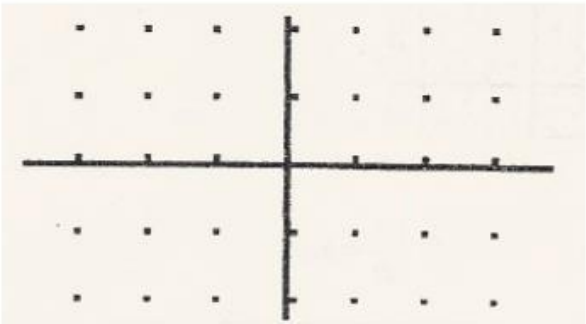
3. $\frac{dy}{dx} = x + y$



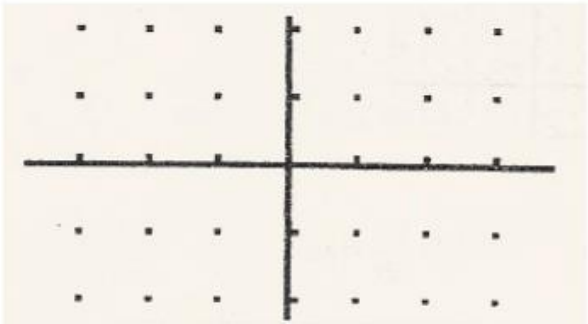
4. $\frac{dy}{dx} = x + 1$



5. $\frac{dy}{dx} = y - 1$



6. $\frac{dy}{dx} = -\frac{y}{x}$

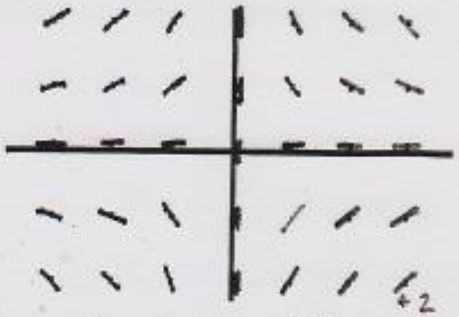


3. $\frac{dy}{dx} = x + y$

1. $\frac{dy}{dx} = x + 1$

5. $\frac{dy}{dx} = y - 1$

6. $\frac{dy}{dx} = -\frac{y}{x}$



When $x = 0$, undef.
$y = 0$, $\frac{dy}{dx} = 0$

Matching

7. $y = 1$

8. $y = x$

9. $y = x^2$

10. $y = \frac{1}{6}x^3$

11. $y = \frac{1}{x^2}$

12. $y = \sin x$

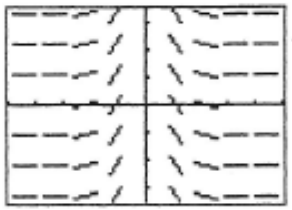
13. $y = \cos x$

14. $y = \ln|x|$

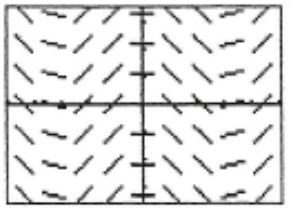
7.D
8.H
9.C
10.F
11.A
12.E
13.B
14.G

Match each slope field with the equation that the slope field could represent.

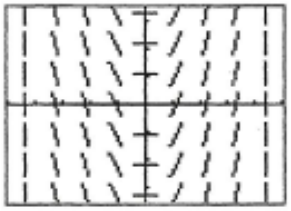
(A)



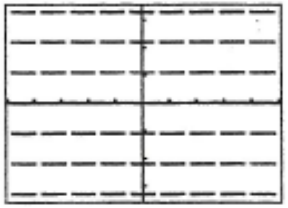
(B)



(C)

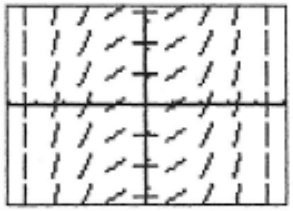


(D)

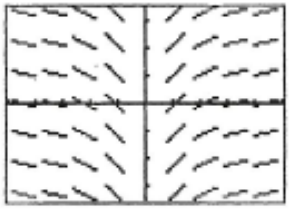


(E)

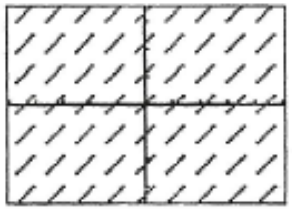
(F)



(G)



(H)

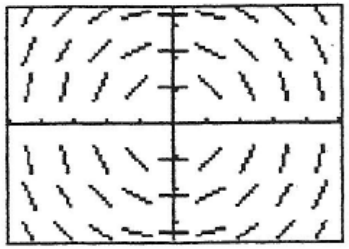


Matching

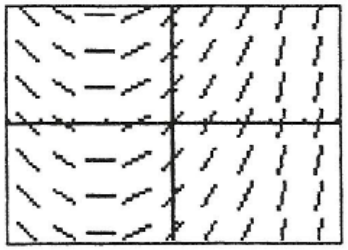
15. B
16.D
17.C
18.A

Match the slope fields with their differential equations.

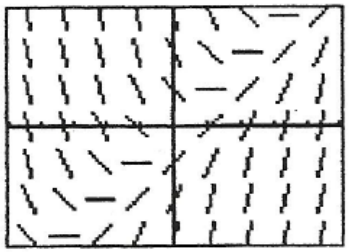
(A)



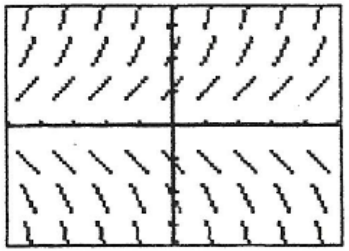
(B)



(C)



(D)



15. $\frac{dy}{dx} = \frac{1}{2}x + 1$

16. $\frac{dy}{dx} = y$

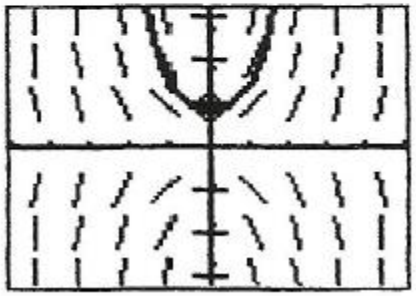
17. $\frac{dy}{dx} = x - y$

18. $\frac{dy}{dx} = -\frac{x}{y}$

19. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.

(a) Sketch the solution curve through the point (0, 2).

(b) Sketch the solution curve through the point $(0, -1)$.



20. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.

(a) Sketch the solution curve through the point (0, 1).

(b) Sketch the solution curve through the point $(-3, 0)$.