Section 10.4 Exponential Growth and Decay

- One model for growth assumes that the population grows at a rate proportional to the size of the population.

$$\frac{dy}{dt} = ky$$

If you solve for k, we can say the relative growth rate is constant.

Let's solve this differential equation.

$$\frac{dy/dt}{y} = k$$

Solve $\frac{dy}{dt} = ky$

$$\frac{dy}{y} = kdt$$

$$\int \frac{dy}{y} = \int kdt$$

$$\ln|y| = kt + C$$

$$e^{\ln|y|} = e^{kt+C}$$

$$|y| = e^{kt} \cdot e^{C}$$

$$y = \pm e^{C} \cdot e^{kt} = Ae^{kt}$$

$$y = Ae^{kt}$$

$$y = Ae^{k\cdot 0}$$

$$y = A$$

$$\boxed{y = y_0 e^{kt}}$$

Let t = 0

Therefore, A = initial population which can denote by y_0

k is the growth constant or relative growth rate

dy/dt is the rate of growth

Exampe

- Given that bacteria grows exponentially, start with 5,000 bacteria and 3 hours later you have 8,000 bacteria. How many will you have in 8 hours from the starting time?

Example

- Carbon-14 has a half-life of 5715 years. How long has something been decaying if 40% of the original Carbon-14 is left?