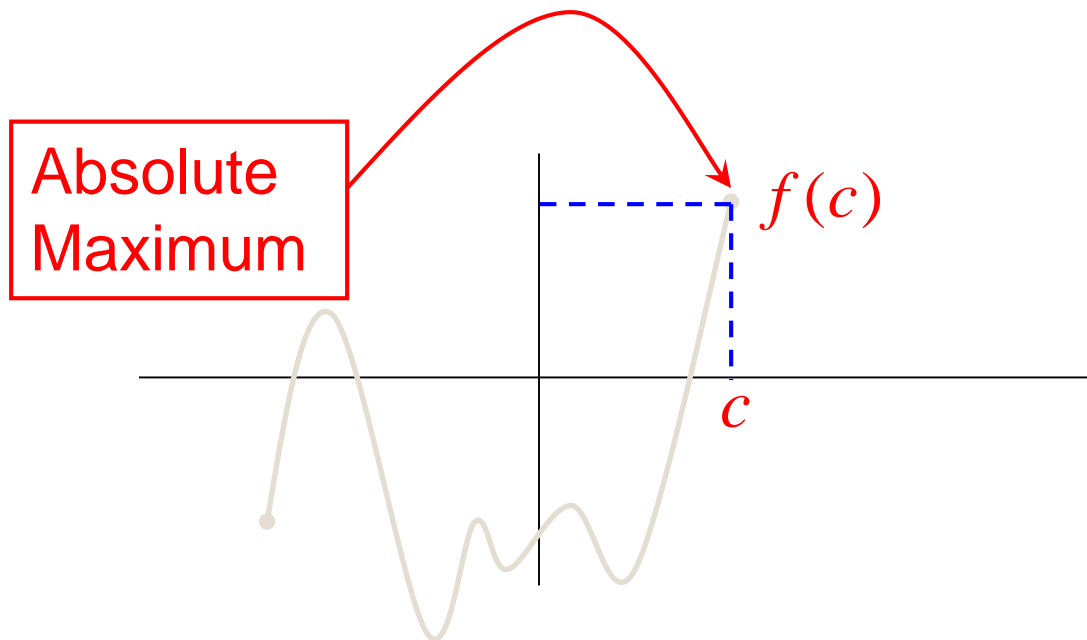


4.1 Maximum and Minimum Values

What is your definition
of a maximum or
minimum value on a
graph?

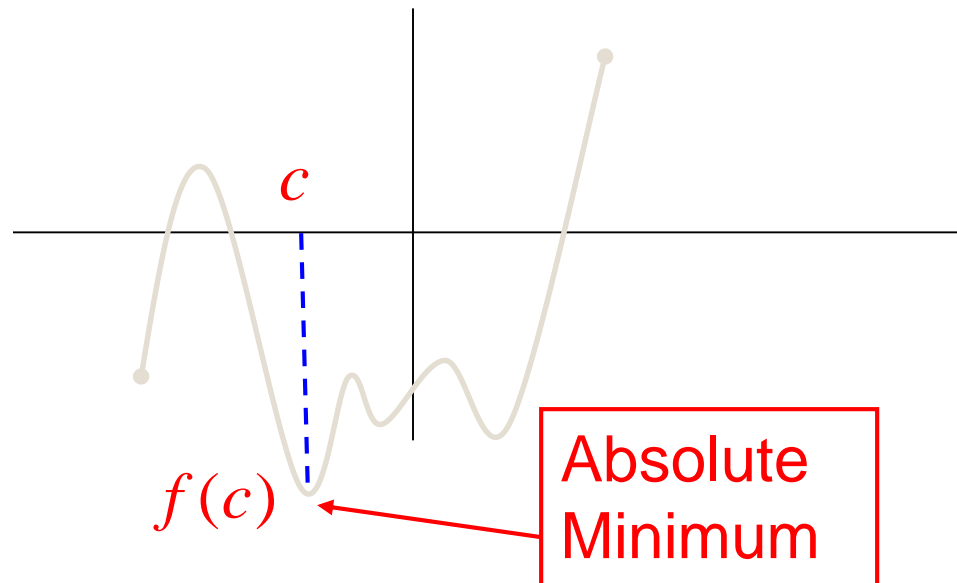
A function f has an **absolute (global) maximum** at $x = c$ if $f(c) \geq f(x)$ **for all** x in the domain D of f .

The number $f(c)$ is called the **absolute maximum** value of f in D



A function f has an **absolute (global) minimum** at $x = c$ if $f(c) \leq f(x)$ **for all** x in the domain D of f .

The number $f(c)$ is called the **absolute minimum** value of f in D

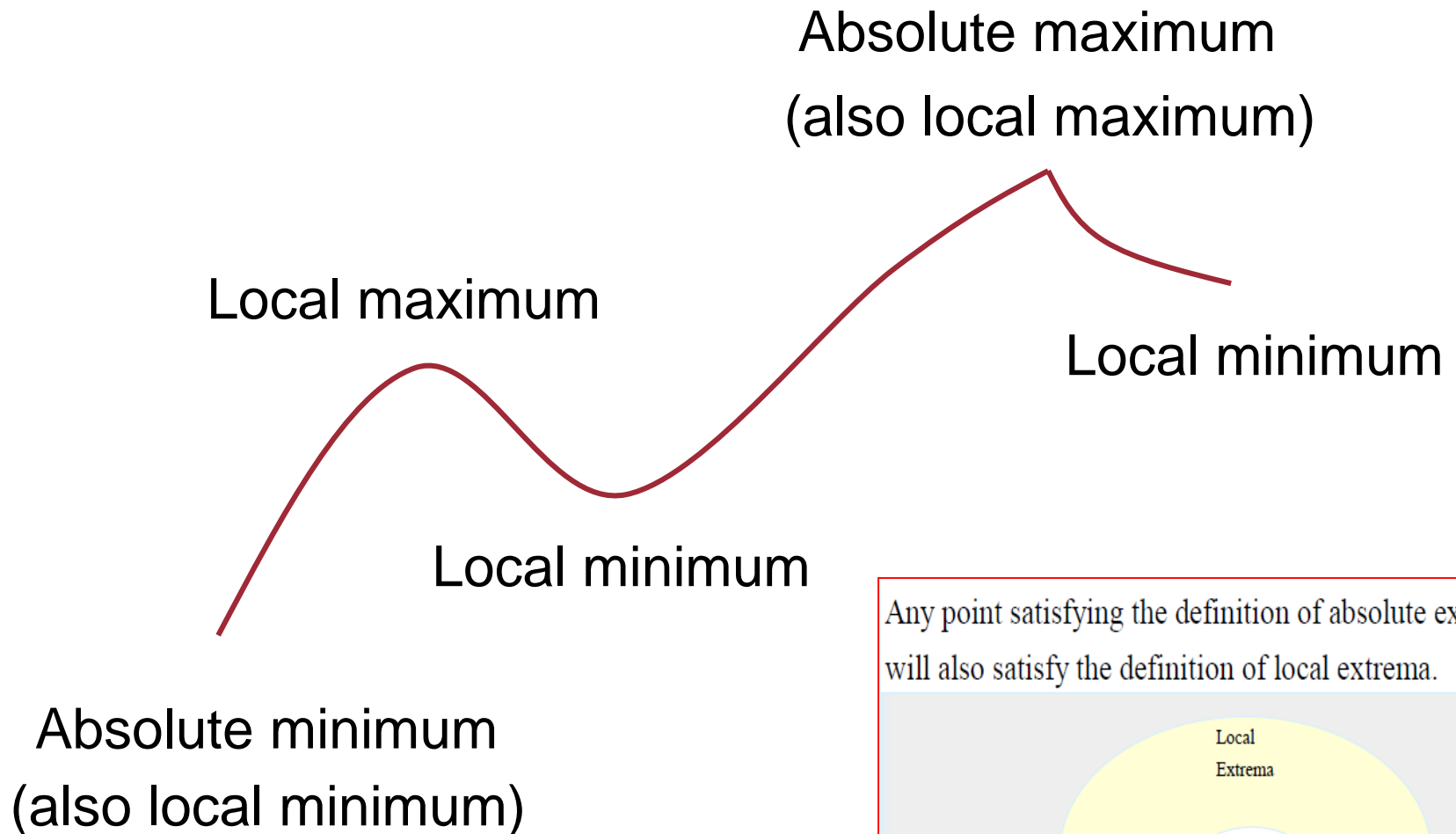


Local Maximum and Minimum

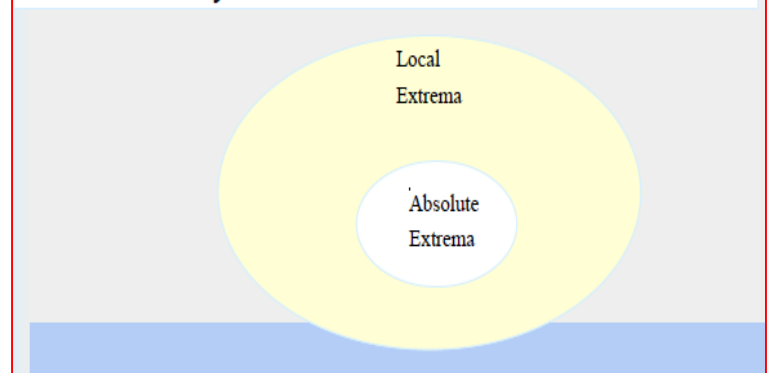
Definition:

- A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c .
- Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c .
- The maximum and minimum values of f are called the **extreme values** of f .

Example:

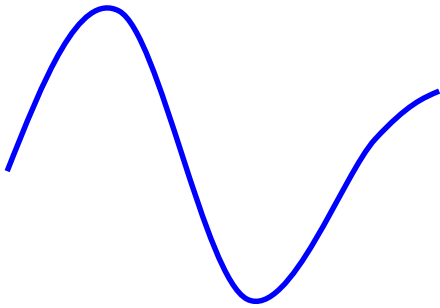


Any point satisfying the definition of absolute extrema will also satisfy the definition of local extrema.

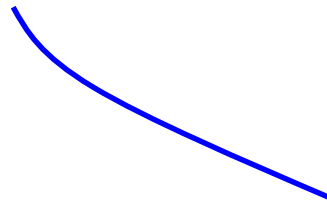


Extreme Value Theorem:

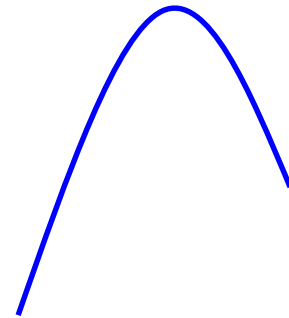
If f is continuous over a closed interval, then f has absolute maximum and minimum over that interval.



Maximum &
minimum
at interior points



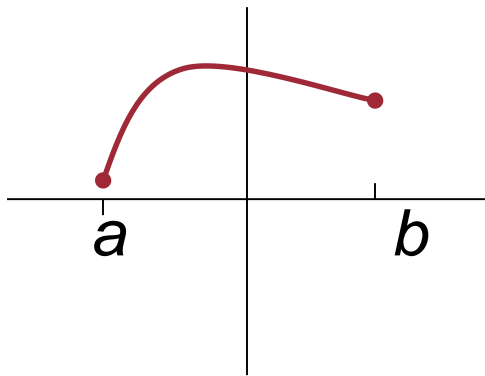
Maximum &
minimum
at endpoints



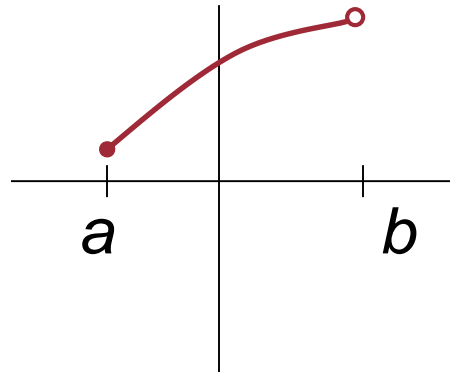
Maximum at
interior point,
minimum at
endpoint

Extreme Value Theorem

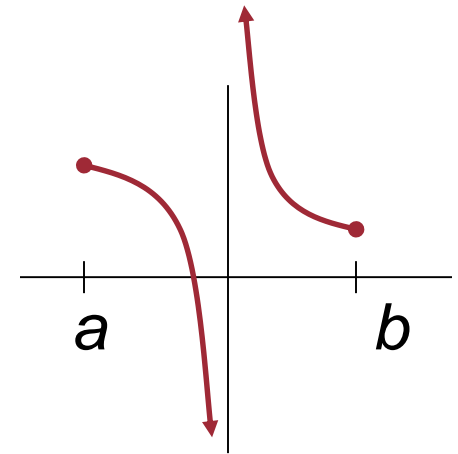
If a function f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum and absolute minimum on $[a, b]$.



Attains max.
and min.



Attains min.
but no max.



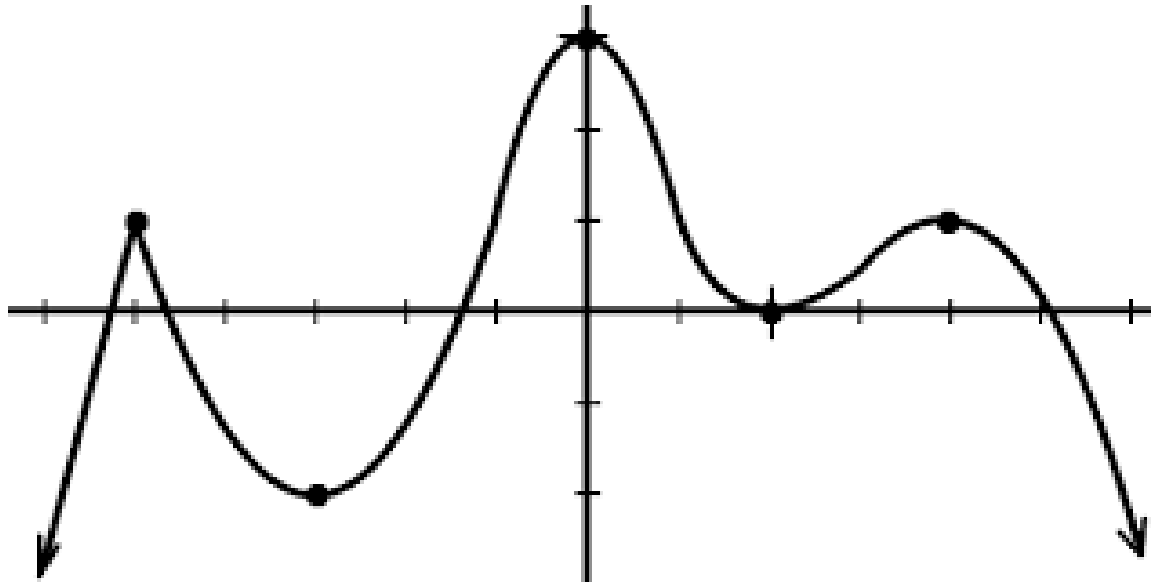
No min. and
no max.

Open Interval

Not continuous

“Where” or “when” means x-value.

“What” means y-value.



The absolute max value is 3,
which occurs at $x = 0$.

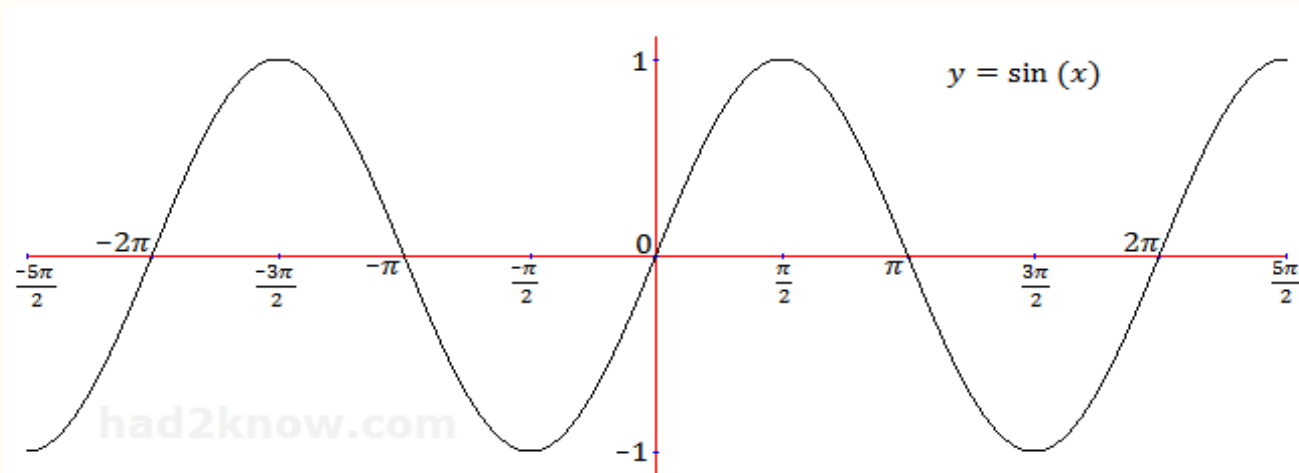
The three local max values are
3 at $x = 0$ and 1 at $x = -5$ and 4.

The two local min values are -2
at $x = -3$ and 0 at $x = 2$.

There is no absolute min
value.

There can be only one absolute min or max value, but it can occur at many locations.

Graph of $y = \sin(x)$



Absolute max of 1 occurs at an infinite number of x-values.

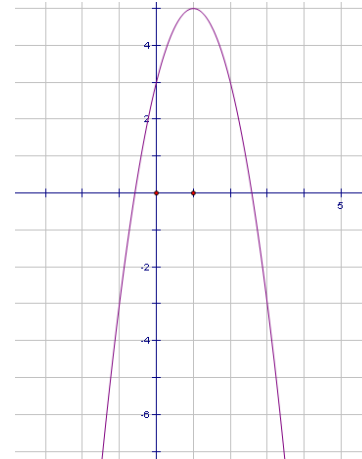
Let's look at $y=x^2 - 6x$ and $y = -2x^2 + 4x + 3$ on the calculator.

Graph:



Use calculator to find the min. It occurs at ?

$$X = 3$$



Use calculator to find the max. It occurs at ?

$$X = 1$$

Use calculator to find $y'(1) = -4$
 $y'(3) = 0$
 $y'(5) = 4$

Use calculator to find $y'(-1) = 8$
 $y'(1) = 0$
 $y'(3) = -8$

2nd Trace, 6: dy/dx

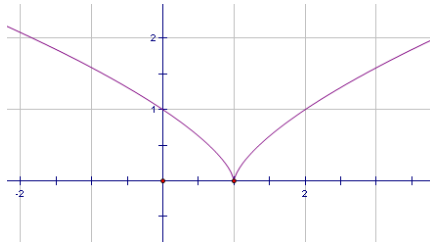
What can you hypothesize about the derivative of a function at a point that is an extrema?

Derivative equals zero.

Sketch the graphs without a calculator.

$$f(x) = (x - 1)^{2/3}$$

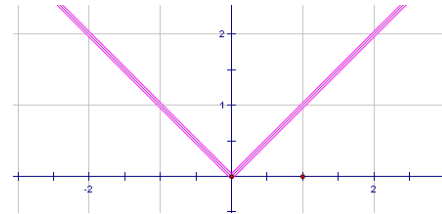
-radical
-even
-shift right 1



$f'(1) = ?$ Does not exist

Extrema? Min at $x = 1$

$$g(x) = |x|$$



$g'(0) = ?$ Does not exist

Extrema? Min at $x = 0$

Conclusion?

Extrema can occur at locations where the derivative does not exist.

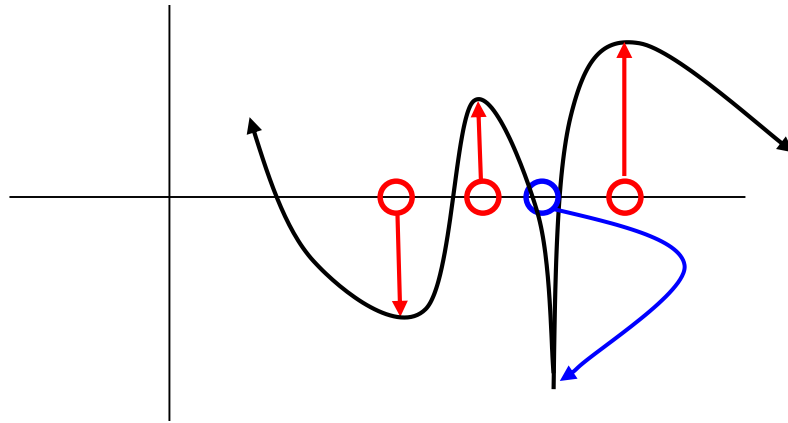
Critical Points of f

A **critical number** of a function f is a number c **in the domain of f** such that

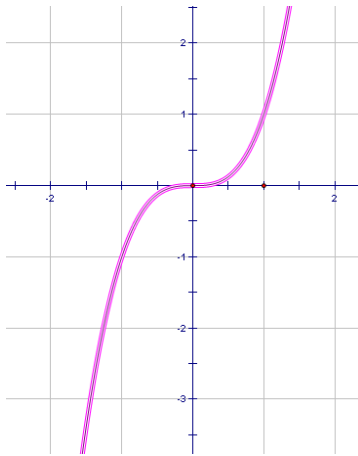
a. $f'(c) = 0$

or

b. $f'(c)$ does not exist



Sketch $f(x) = x^3$.



$$f'(0) = ? \quad 0$$

But the function does not have any extrema.


Find $f'(-2)$ and $f'(2)$ Both equal 12


The slopes stay positive.


Conclusion?

Graph needs to change direction to create an extreme value.

Increasing/Decreasing/Constant

 If $f'(x) > 0$ for each value of x in an interval (a, b) , then f is increasing on (a, b) .

 If $f'(x) < 0$ for each value of x in an interval (a, b) , then f is decreasing on (a, b) .

 If $f'(x) = 0$ for each value of x in an interval (a, b) , then f is constant on (a, b) .

1st Derivative Test

- Find the critical points
 - Set derivative = 0
 - Where is derivative undefined

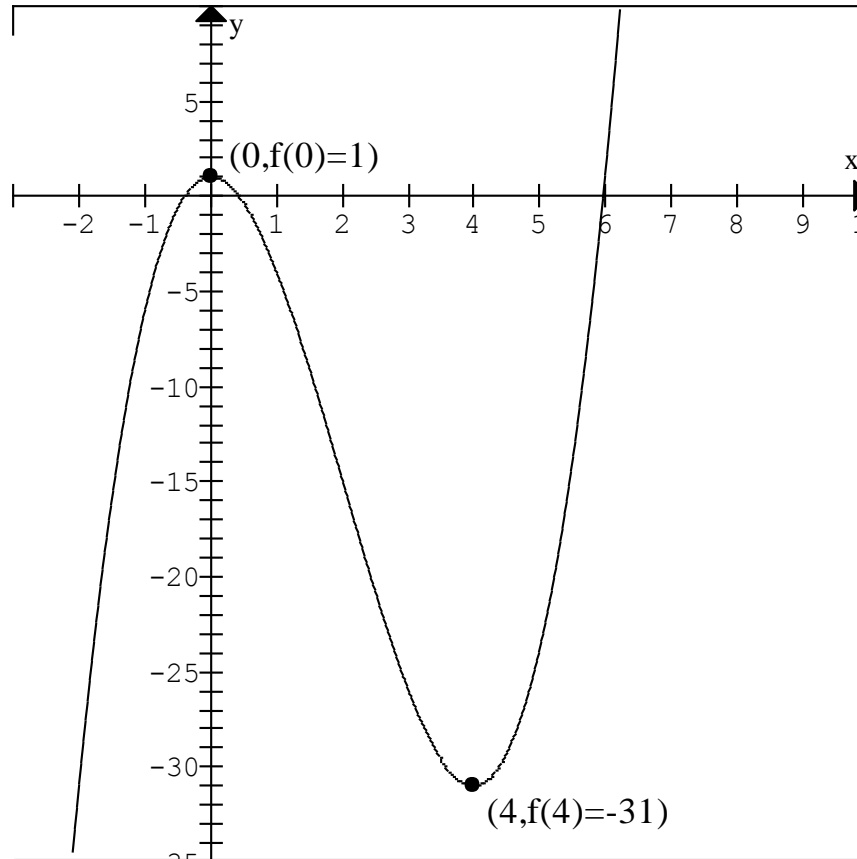
- Analyze sign behavior
 - Create a sign chart with the critical points
 - Test #'s and see if you get + or - results

- Interpret the results
 - Where is function increasing/decreasing
 - Where are extrema
 - What type of extrema

The First Derivative Test

Find all the relative extrema of $f(x) = x^3 - 6x^2 + 1$.

Graph of $f(x) = x^3 - 6x^2 + 1$.



Another Example

Find all the relative extrema of $f(x) = \sqrt[3]{x^3 - 3x}$.

$$f'(x) = \frac{x^2 - 1}{\sqrt[3]{(x^3 - 3x)^2}}$$

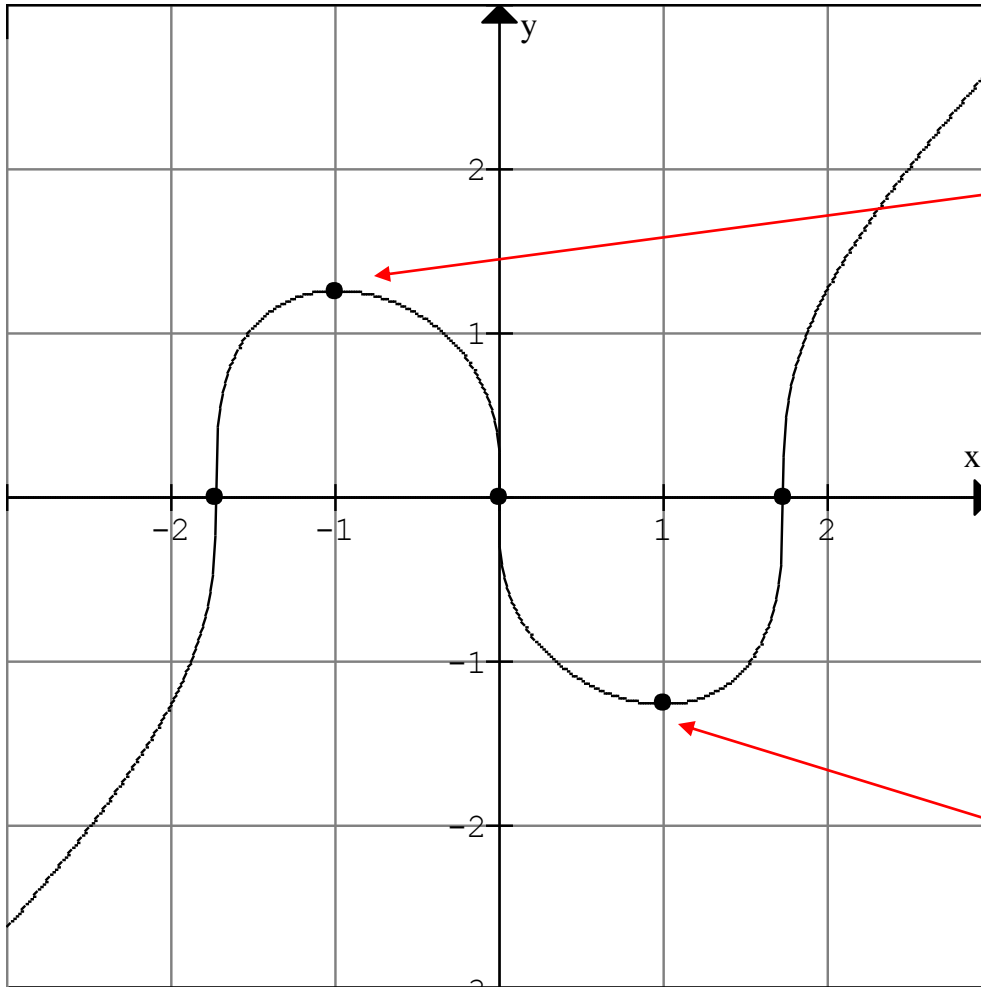
$$f'(x) = 0: \quad x = \pm 1$$

$$f'(x) \text{ is undefined:} \quad x = 0, \pm \sqrt{3}$$

$$f'(x) = 0: x = \pm 1$$

$$F'(x) \text{ is undefined: } x = 0, \pm \sqrt{3}$$

Graph of $f(x) = \sqrt[3]{x^3 - 3x}$.



Local max. $f(-1) = \sqrt[3]{2}$

Local min. $f(1) = -\sqrt[3]{2}$

Another Example

Find all the relative extrema of $y = x^2 e^x$

On a closed interval you must consider endpoints as well as critical numbers.

- Find the absolute min and max of $f(x) = x^2 - 10x + 27$ on the interval $[1, 6]$.

Find all local and absolute extrema for

$$y = \frac{x^3}{3} - 3x^2 + 8x \quad 0 \leq x \leq 10$$

Find all local and absolute extrema for

$$y = \frac{1}{x} + \ln x$$

$$\frac{1}{2} \leq x \leq 4$$

Find all local and absolute extrema for

$$y = x^{2/3}$$

$$-1 \leq x \leq 8$$