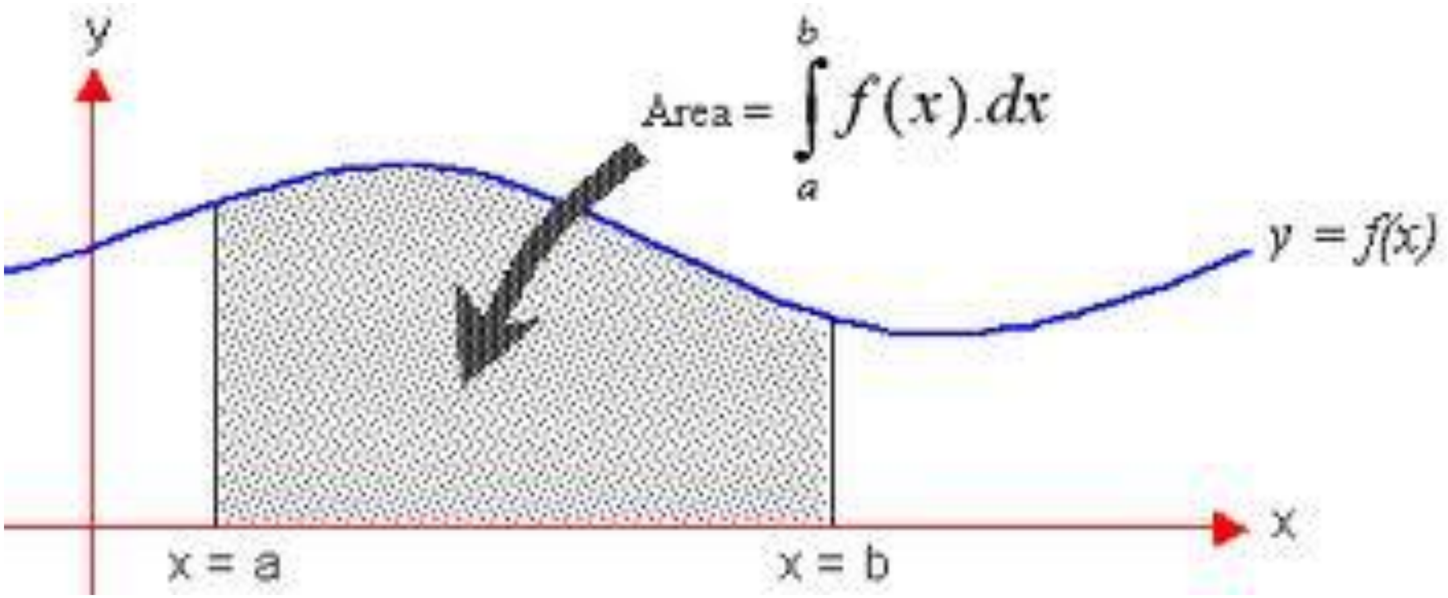
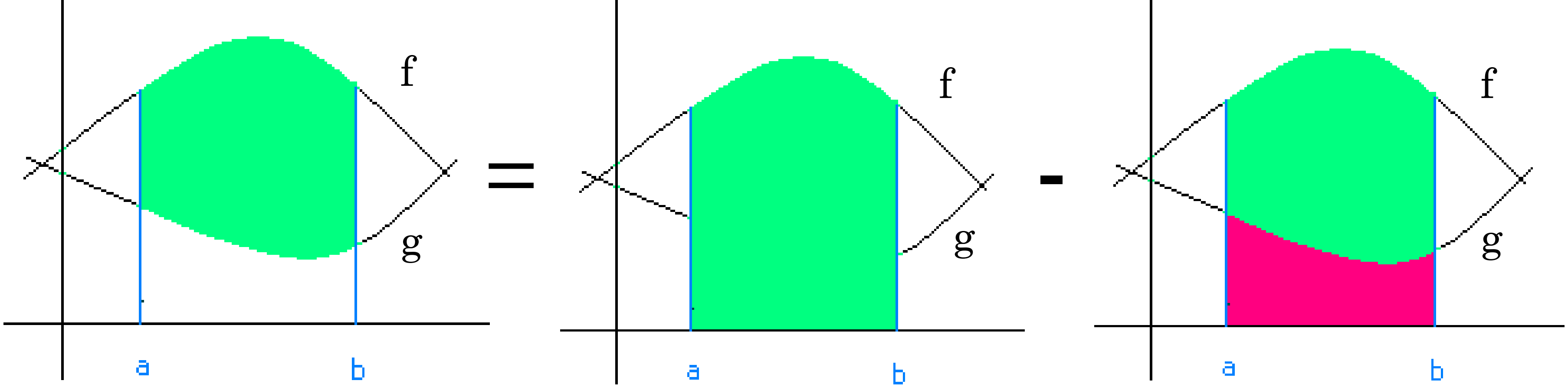
Section 6.1 Area between Curves

WE HAVE FOUND AREA UNDER A CURVE USING INTEGRATION

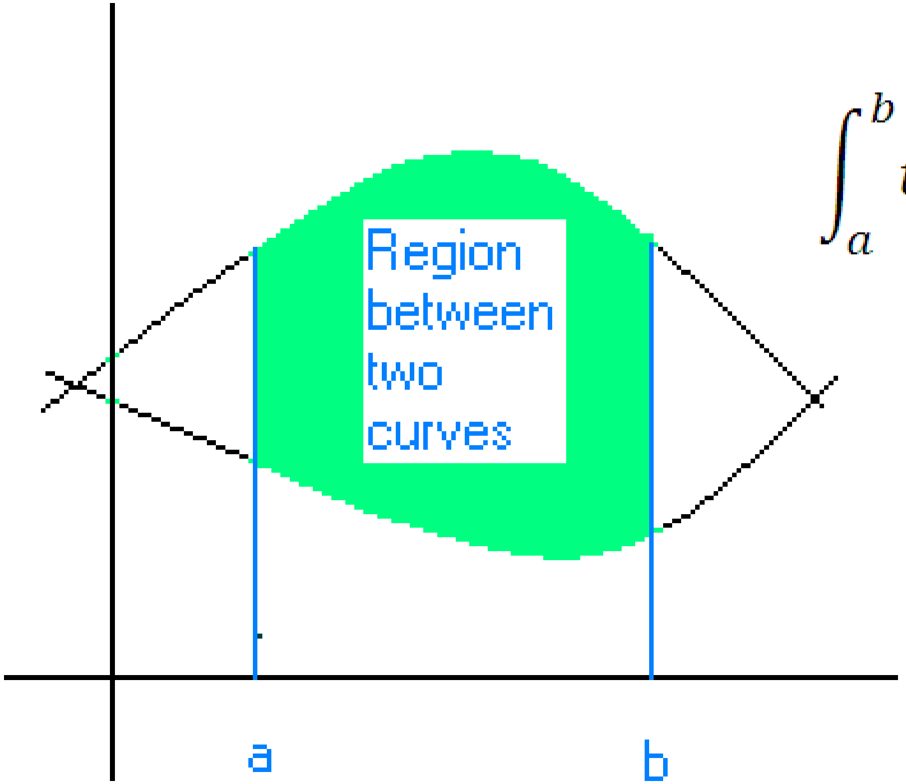


Today we will find the area between curves



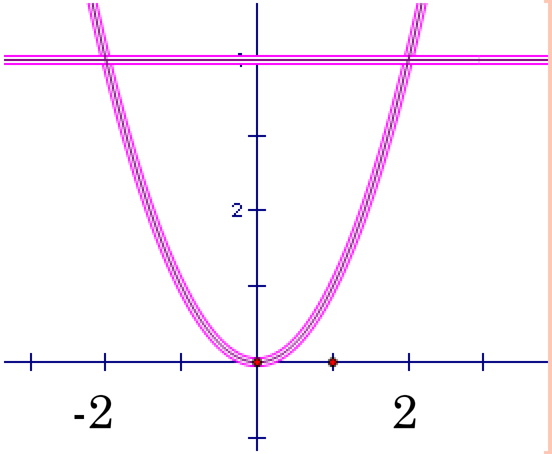
Area of region between f and g = Area of region under f(x) - Area of region under g(x)

$$\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$



$$\int_a^b top\ curve - bottom\ curve\ dx$$

EXAMPLE: FIND THE AREA OF THE REGION BOUND BY Y = 4 AND Y = X^2



Step 1: Sketch graph of region

Step 2: Set functions equal to each other to find intersection points

$$4 = x^2$$
$$\pm 2 = x$$

Step 3: Set up integral

$$\int_a^b top\ curve - bottom\ curve\ dx$$

$$\int_{-2}^{2} 4 - x^2\, dx$$

Step 4: Evaluate using calculator

Area = 10.667

You try:

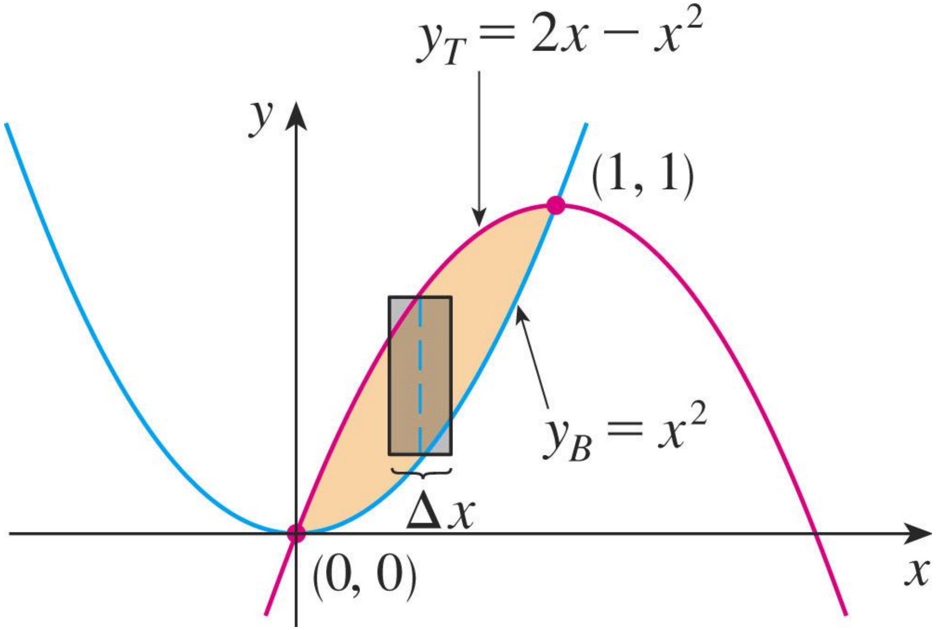
Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

From the figure, we see that the top and bottom boundaries are:

$$y_T = 2x - x^2 \qquad \text{and} \qquad y_B = x^2$$

$x^2 = 2x - x^2$, or $2x^2 - 2x = 0$.

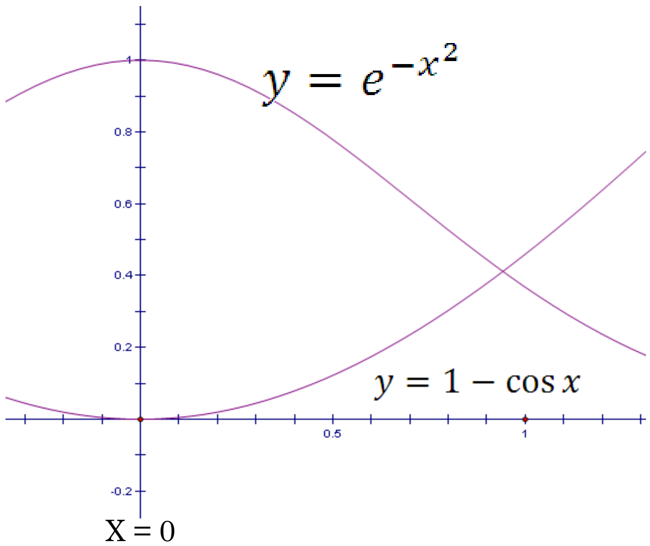
Thus, $2x(x - 1) = 0$,
so $x = 0$ or 1.



$$A = \int_0^1 2x - 2x^2 \; dx = 2\int_0^1 x - x^2 \; dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$

Find area of region bound by y = 1- cos x, $y = e^{-x^2}$, and x = 0.



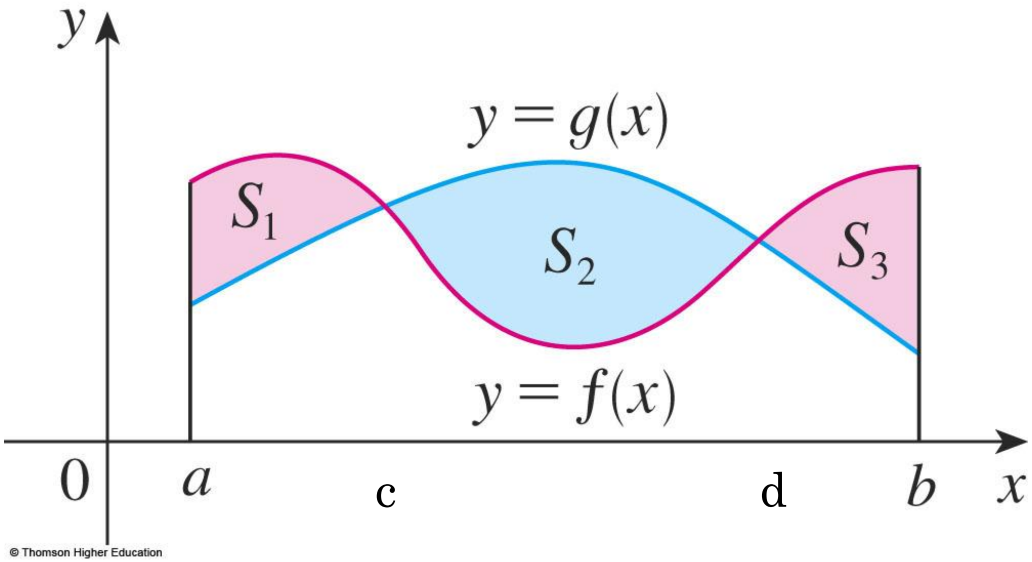
Find the intersection point using your calculator. It will be stored as letter x.

X = .94194408

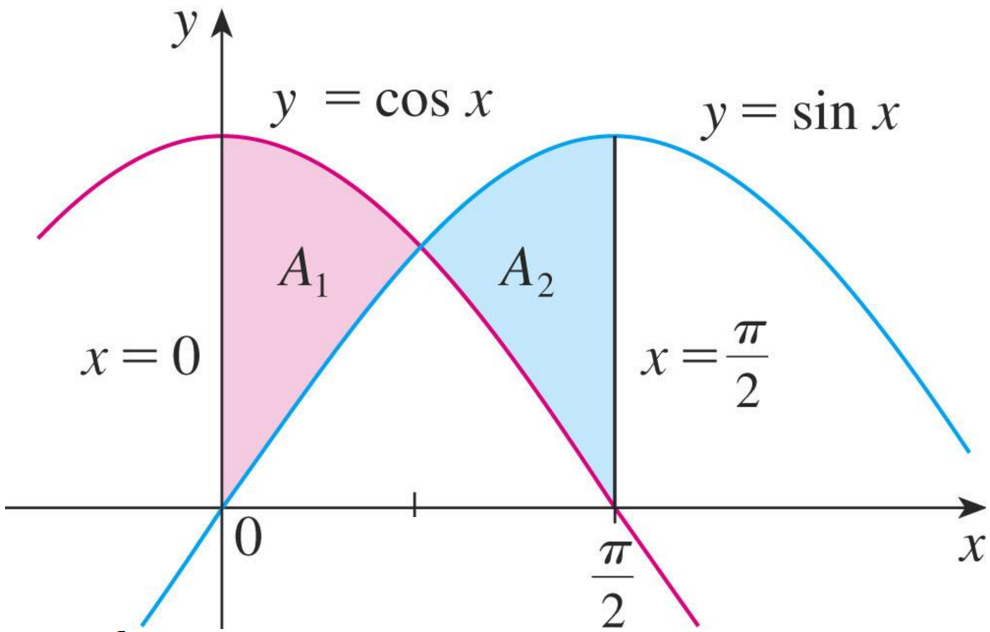
$$\int_0^x e^{-x^2} - (1 - \cos x)\ dx = 0.591$$

To find the area between f(x) and g(x) on the interval from a to b, you must split the area into 3 regions.

$$\int_a^c f(x) - g(x)dx + \int_c^d g(x) - f(x)dx + \int_d^b f(x) - g(x)dx$$



Example: find the area of the region bound by y = sin x, y = cos x, x = 0 and x = π/2



1. Sketch graph.

2. Find intersection point.

   sin x = cos x

   x = π/4

3. Set up integral.

$$\int_0^{\pi/4} \cos x - \sin x \, dx + \int_{\pi/4}^{\pi/2} \sin x - \cos x \, dx$$

Or if you notice symmetry

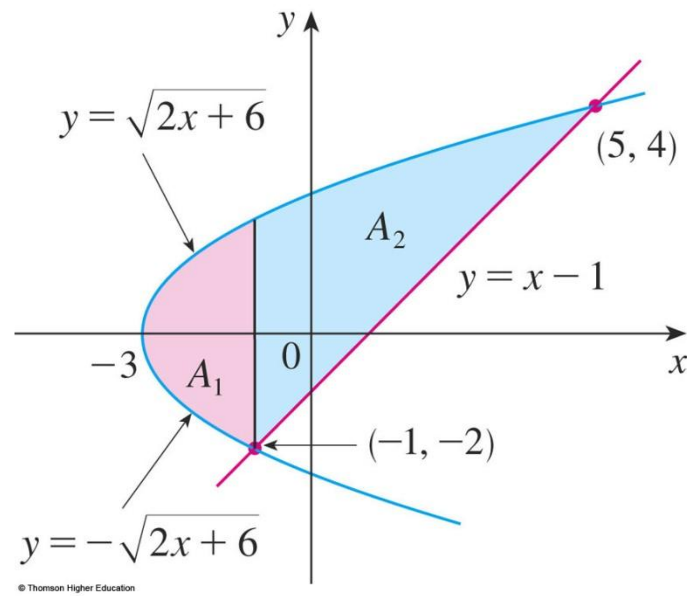
$$2\int_0^{\pi/4} \cos x - \sin x \, dx$$

Area = .828

FIND THE AREA ENCLOSED BY THE LINE $Y = X - 1$ AND THE PARABOLA $Y^2 = 2X + 6$.

1. Sketch graph and find the intersection pts.

   $y^2 = 2x + 6$ is not a function, need to solve for y and graph each function separately.



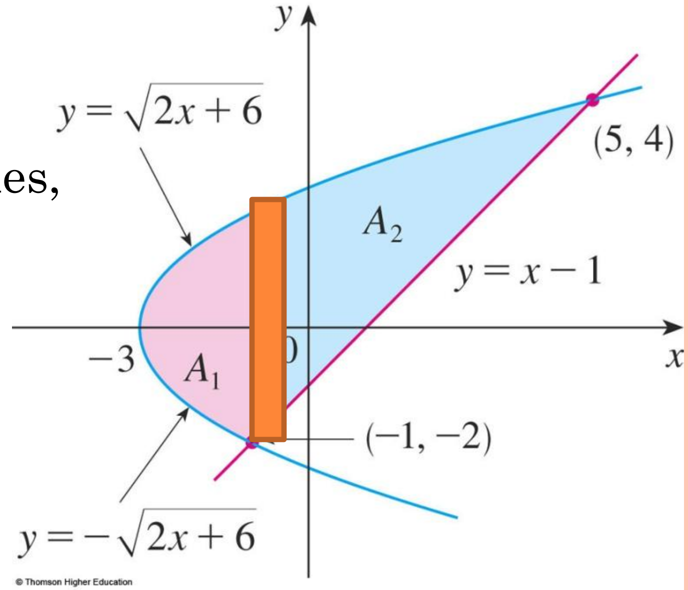
2. Set up integral.

$$\int_{-3}^{-1} \sqrt{2x+6} - \left(-\sqrt{2x+6}\right) dx + \int_{-1}^{5} \sqrt{2x+6} - (x-1)\, dx$$

Area = 18

We calculated the area based on vertical rectangles,

$$\int_{x-value}^{x-value} top\ curve - bottom\ curve\ dx$$

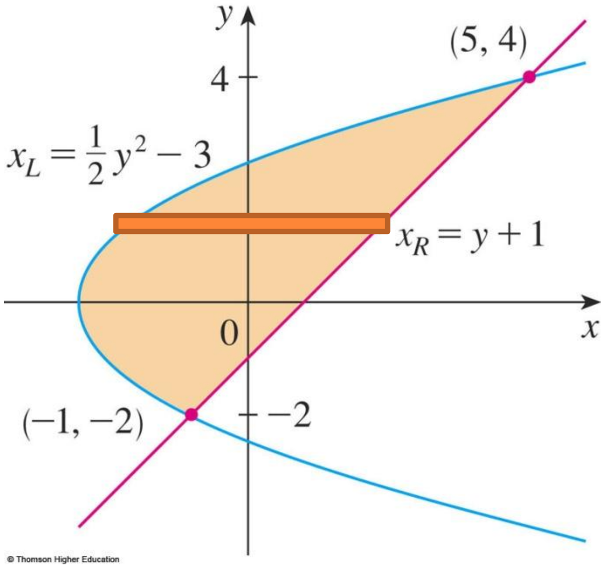


Sometimes it is easier to calculate the area using horizontal rectangles.

Notice the equations have been written in terms of y.

$$\int_{y-value}^{y-value} right\ curve - left\ curve\ dy$$

$$\int_{-2}^{4} y + 1 - (\frac{1}{2}y^2 - 3)\ dy \quad =18$$

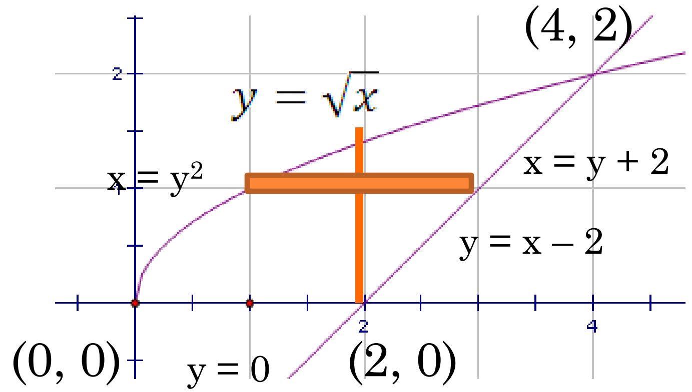


Try Finding the Area Both Ways

Find the area of the region bound by y = x – 2 , $y = \sqrt{x}$, and the x-axis.

Top – Bottom

$$\int_0^2 \sqrt{x} - 0\, dx + \int_2^4 \sqrt{x} - (x-2)dx$$



Right – Left

Right : x = y + 2, Left: x = y^2

Area = 3.333

$$\int_0^2 y + 2 - y^2 dy$$

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