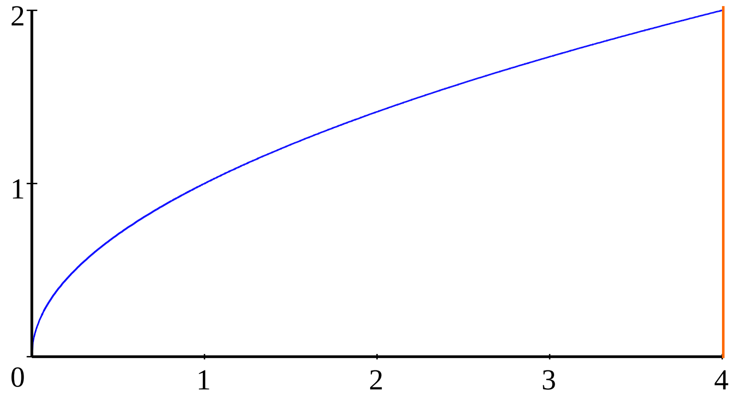
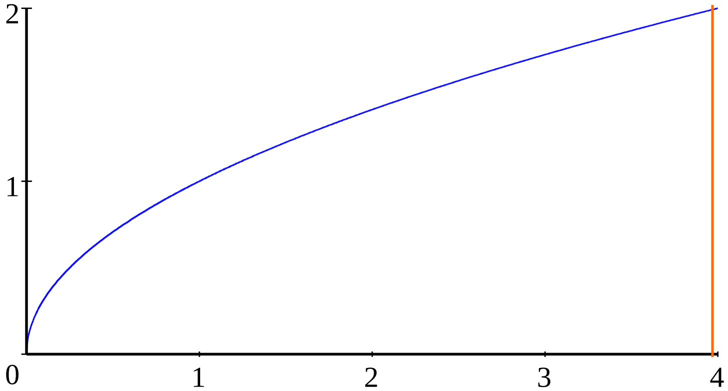
Section 6.2 Volume of solids of revolution

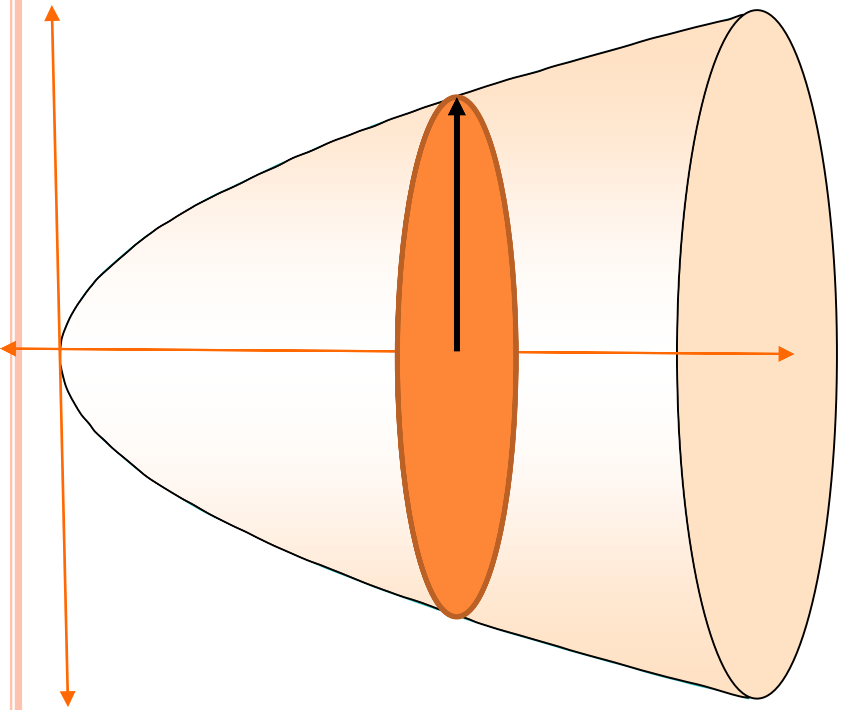
- A solid of revolution is a solid that is generated by revolving a plane region about a line.

- Let our region be enclosed by $y = \sqrt{x}$, y = 0 and x = 4.



Let's rotate the region about the x-axis to create a solid.





What shape are the cross sections?

Circles

What is the radius of the circles?

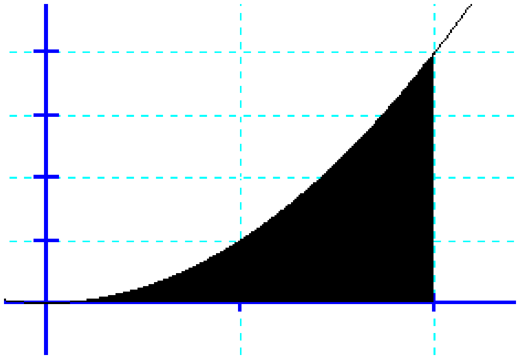
Radius = y = $\sqrt{x}$

Volume would be the sum of all the circle from 0 to 4.

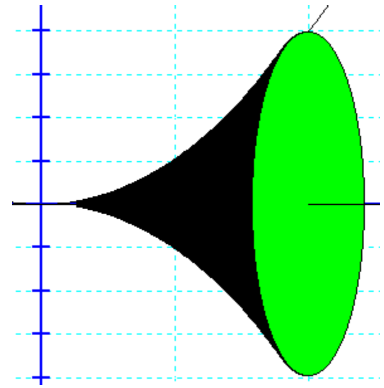
$$V = \int_0^4 \pi r^2 dx = \int_0^4 \pi(\sqrt{x})^2 dx = \pi \int_0^4 x dx$$

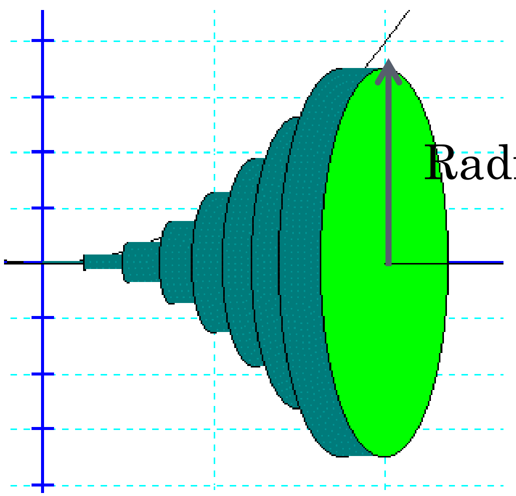
$$\text{Volume} = \frac{\pi}{2} x^2 \Big|_0^4 = 8\pi \ u^3$$

Revolve the region bound by $y = x^2$, y = 0 and x = 2 about the x-axis.



What would the solid look like?





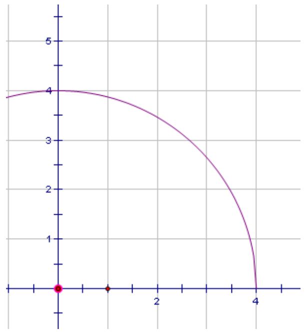
Radius = $y = x^2$

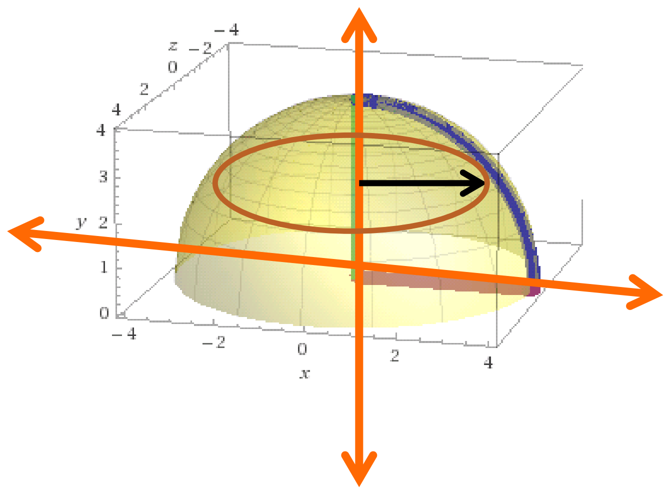
To find the volume add the circle from x = 0 to x = 2

$$\int_0^2 \pi r^2 dx$$

$$V = \int_0^2 \pi (x^2)^2 dx$$ = 6.4π cubic units

Revolve the region bound by $y = \sqrt{16 - x^2}$, y = 0, and x = 0 about the y-axis





When rotating around a vertical line, you are adding circles stacked vertically from y-value to y-value.

This solid is a sum of circles from y = 0 to y = 4.

$$Volume = \int_0^4 \pi r^2 dy$$

$$= \int_0^4 \pi\left(\sqrt{16 - y^2}\right)^2 dy$$

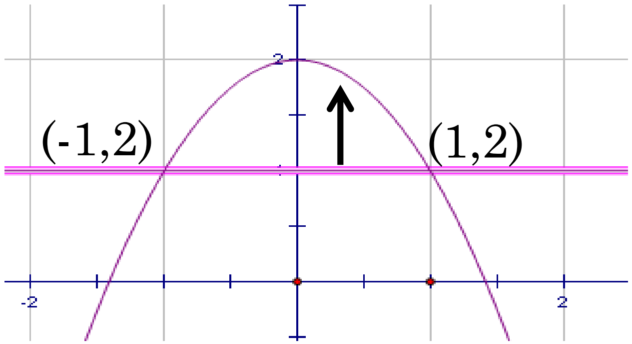
$$= \pi \int_0^4 16 - y^2 \, dy$$

$$= \frac{128}{3}\pi \ units^3$$

r = x = f(y)

$$y = \sqrt{16 - x^2}$$
$$y^2 = 16 - x^2$$
$$x^2 = 16 - y^2$$
$$x = \sqrt{16 - y^2}$$

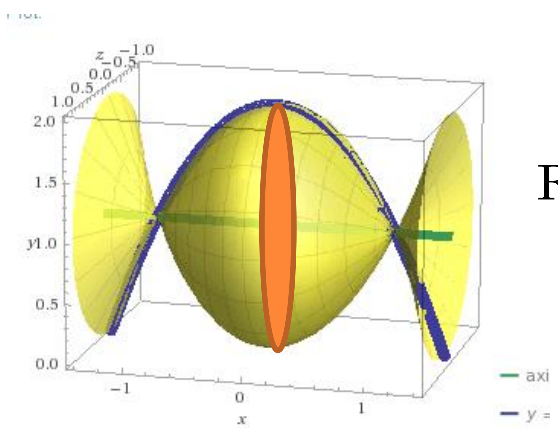
Find the volume of the solid generated by revolving the region defined by $y = 2 - x^2$, and y = 1, about y = 1.



The volume is the sum of circles from x-value to x-value.

$$2 - x^2 = 1$$
$$x^2 = 1$$
$$x = 1, -1$$

$$\text{Volume} = \int_{-1}^{1} \pi r^2 dx$$



Radius is top curve – bottom curve

Radius = $(2 - x^2) - 1$
$= 1 - x^2$

$$\int_{-1}^{1} \pi (1 - x^2)^2 dx$$

$$= \frac{16}{15}\pi \ units^3$$

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