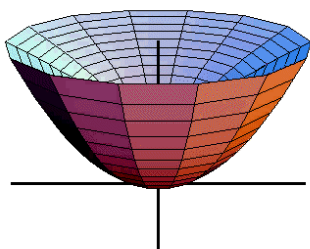
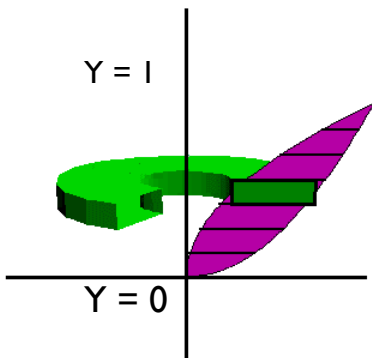
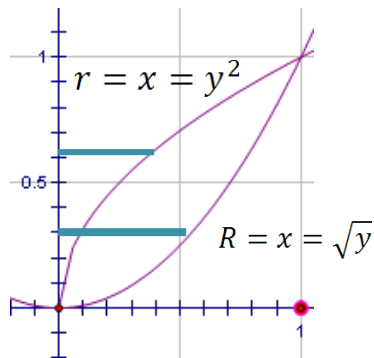




Section 7.3 Volume by Shells

Rotate the region bound by $y = \sqrt{x}$ and $y = x^2$ about the y -axis.

Find Volume.

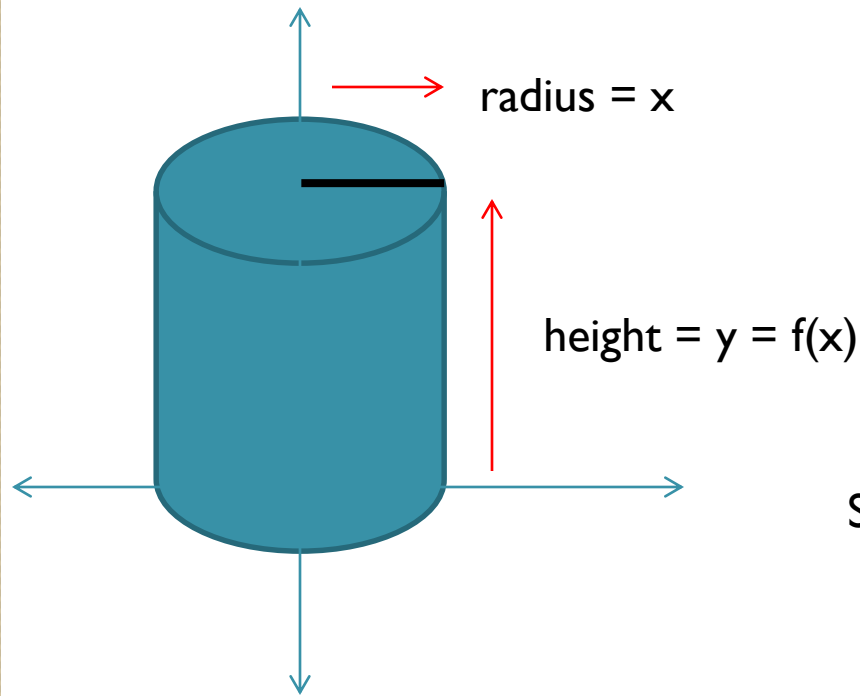


Washer Method: $\int_c^d \pi R^2 - \pi r^2 dy$

$$\int_0^1 \pi (\sqrt{y})^2 - \pi (y^2)^2 dy$$

$$\pi \int_0^1 y - y^4 dy = 0.3\pi$$

The washer method adds infinite washers.
The shell method adds the surface area of infinite cylinder layers.



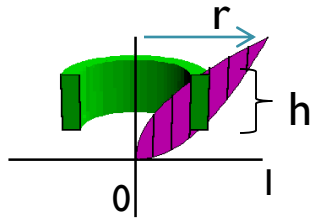
$$\text{Surface Area} = 2\pi r h$$

$$\text{Shell Method: } \int_a^b 2\pi r h dx$$

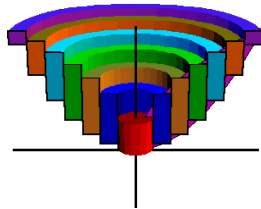
Shell Method



Let's rotate the region bound by $y = \sqrt{x}$ and $y = x^2$ about the y-axis again. But, let's use the shell method to find volume.



$$\int_a^b 2\pi r h dx$$

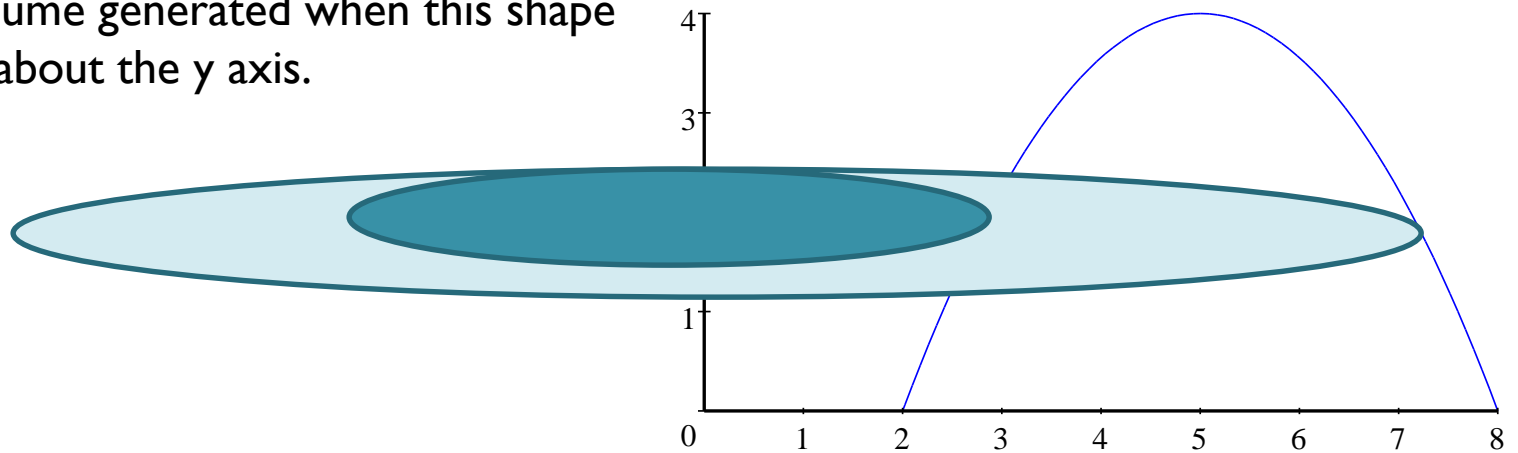


$$r = x$$

$$h = \text{top} - \text{bottom} = \sqrt{x} - x^2$$

$$\int_0^1 2\pi x(\sqrt{x} - x^2) dx = 0.3\pi$$

Find the volume generated when this shape is revolved about the y axis.



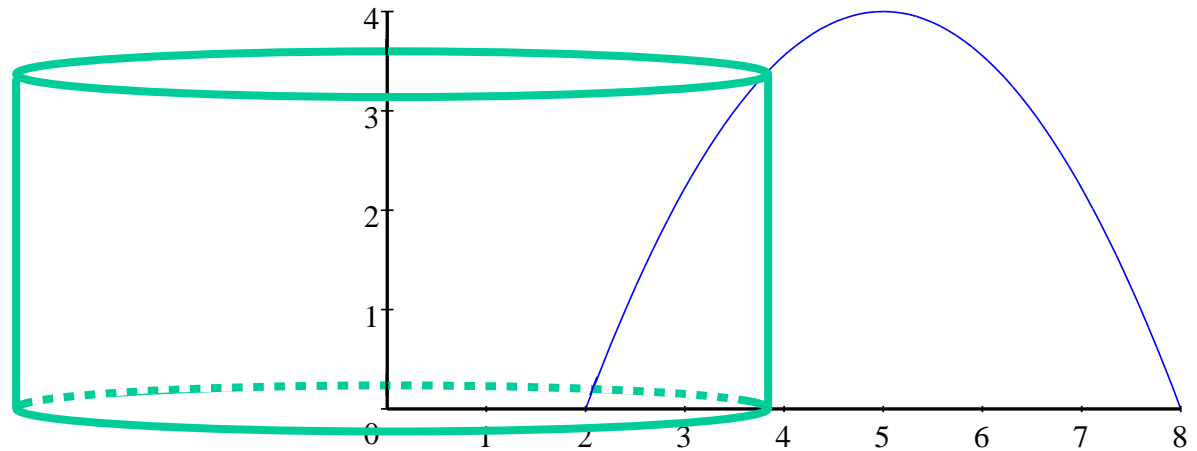
$$y = -\frac{4}{9}(x^2 - 10x + 16)$$



We can't solve for x, so we can't use a horizontal slice directly.



If we take a vertical slice
and revolve it about the
 y -axis
we get a cylinder.



$$y = -\frac{4}{9}(x^2 - 10x + 16)$$

Shell method:

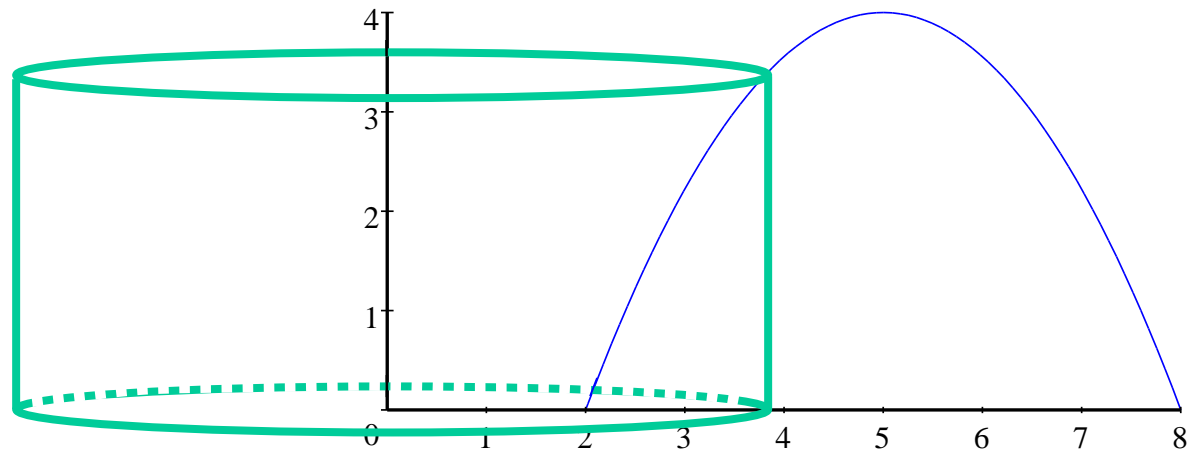
Lateral surface area of cylinder

= circumference \cdot height

$$= 2\pi r \cdot h$$

Volume of thin cylinder = $2\pi r \cdot h \cdot dx$





Volume of thin cylinder = $2\pi r \cdot h \cdot dx$

$$y = -\frac{4}{9}(x^2 - 10x + 16)$$

$$\int_2^8 2\pi x \left[-\frac{4}{9}(x^2 - 10x + 16) \right] dx$$

$$= 160\pi$$

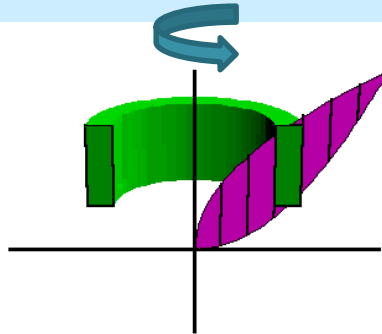
$$\approx 502.655 \text{ cm}^3$$

$\underbrace{2\pi x}_r$ (circumference) $\underbrace{\left[-\frac{4}{9}(x^2 - 10x + 16) \right]}_h$ (height) $\underbrace{dx}_{\text{thickness}}$

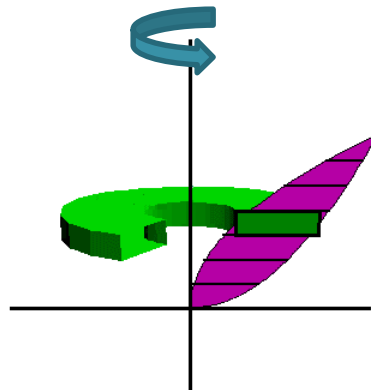
Note: When entering this into the calculator, be sure to enter the multiplication symbol before the parenthesis.



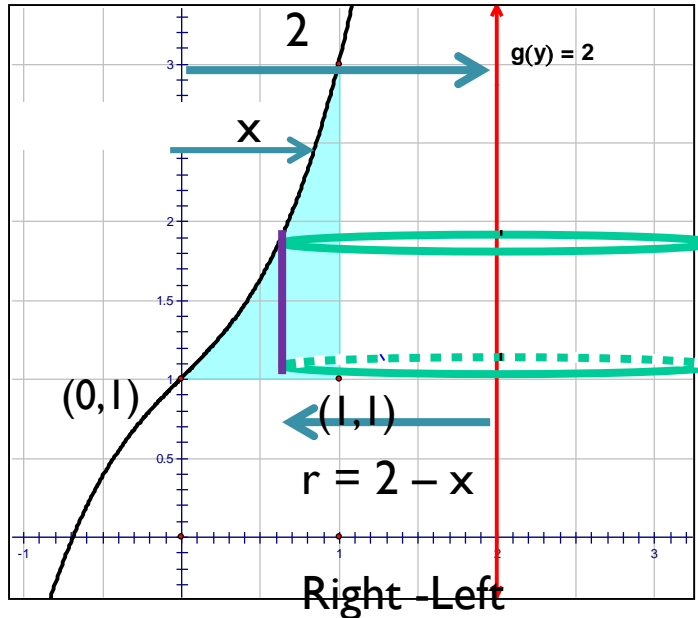
When the strip is parallel to the axis of rotation, use the shell method.



When the strip is perpendicular to the axis of rotation, use the washer method.



Find the volume of the solid of revolution formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, $x = 1$ about the line $x = 2$.



Take a vertical slice.

Rotate it around $x = 2$, to create a cylinder.

$$h = x^3 + x + 1 - 1$$

Top - Bottom

Limits of integration: 0 to 1

$$\text{Volume} = \int_a^b 2\pi r h dx$$

$$V = 2\pi \int_0^1 \left[(2 - x)(x^3 + x + 1 - 1) \right] dx$$

$$= 2\pi \int_0^1 (-x^4 + 2x^3 - x^2 + 2x) dx = \frac{29\pi}{15}$$

Practice:

p. 427 # 7 – 21 odd