Section 8.7 Indeterminate Forms

Review: $\lim_{x \to 2} \frac{x^2 + 6x - 16}{x - 2}$

$$= \frac{2^2 + 6 \cdot 2 - 16}{2 - 2} = \frac{0}{0}$$

We end up with an indeterminate form

Note <u>why</u> this is indeterminate

$$\frac{0}{0} = n \quad \Rightarrow 0 \cdot n = 0 \quad n = ?$$

$$\lim_{x \to 2} \frac{x^2 + 6x - 16}{x - 2}$$

We can evaluate it by factoring and canceling:

$$= \lim_{x \to 2} \frac{(x - 2)(x + 8)}{x - 2}$$

$$= \lim_{x \to 2} x + 8 = 2 + 8 = 10$$

Review: $$\lim_{x\to\infty}\frac{3x^2-1}{2x^2+1}=\frac{3(\infty)^2-1}{2(\infty)^2+1}=\frac{\infty}{\infty}$$

We end up with another indeterminate form

$$\lim_{x\to\infty}\frac{3x^2-1}{2x^2+1}$$

Divide numerator and denominator by x^2 (highest power in the denominatior)

$$=\lim_{x\to\infty}\frac{3-(1/x^2)}{2+(1/x^2)}=\frac{3-0}{2+0}=\frac{3}{2}$$

Or use L'Hôpital's Rule

If we have an indeterminate form of type $\frac{0}{0}$ or ∞/∞,

then $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$

$$\lim_{x \to 2} \frac{x^2 + 6x - 16}{x - 2} = \lim_{x \to 2} \frac{2x + 6}{1} = \frac{2 \cdot 2 + 6}{1} = 10$$

$$\lim_{x \to \infty} \frac{3x^2 - 1}{2x^2 + 1} = \lim_{x \to \infty} \frac{6x}{4x} = \frac{6}{4} = \frac{3}{2}$$

Try: $\lim_{x\to 1} \frac{\ln x}{x-1} = \frac{0}{0}$

$$\lim_{x\to 1} \frac{\ln x}{x-1} = \lim_{x\to 1} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)} = \lim_{x\to 1} \frac{1/x}{1} = \lim_{x\to 1} \frac{1}{x} = 1$$

Try: $$\lim_{x\to\infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{e^x}{2} = \frac{\infty}{2} = \infty$$

Remember that L'Hôpital's Rule can be applied only to quotients leading to the indeterminate forms $\frac{\infty}{\infty}$ *or* $\frac{0}{0}$

There are similar forms that you should recognize as "determinate."

$$\frac{\#}{\infty} \to 0$$

$$\frac{\infty}{\#} \to \pm\infty$$

$$\frac{\#}{0} \to \pm\infty$$

$$\infty + \infty \to \infty$$

$$-\infty - \infty \to -\infty$$

$$0^{\infty} \to 0$$

$$0^{-\infty} \to \infty$$

$$\frac{0}{0} \qquad \frac{\infty}{\infty} \qquad \infty-\infty \qquad 1^{\infty} \qquad 0^{0} \qquad \infty^{0} \qquad 0\cdot\infty$$

have been identified as *indeterminate*.

The first two can be evaluated using L'Hôpital directly.

$\infty - \infty \; and \; 0 \cdot \infty$ need to be rewritten as a fraction in order to use L'Hôpital.

We will deal with the indeterminate powers tomorrow.

$$\lim_{x \to 0+} x \ln x$$

$$\lim_{x \to \frac{\pi}{2}^-} (\sec x - \tan x)$$

Try:

$$\lim_{x \to \pi -} (x - \pi) \cot x$$ =1

$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$$ =1/2

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$$ ← This is indeterminate form $\infty-\infty$

Rewrite as a ratio!

If we find a common denominator and subtract, we get:

$$\lim_{x\to 1}\left(\frac{x-1-\ln x}{(x-1)\ln x}\right)$$ ← Now it is in the form $\frac{0}{0}$

$$\lim_{x\to 1}\left(\frac{1-\frac{1}{x}}{\frac{x-1}{x}+\ln x}\right)$$ ← L'Hôpital's rule applied once.

$$\lim_{x\to 1}\left(\frac{x-1}{x-1+x\ln x}\right)$$ ← Fractions cleared. Still $\frac{0}{0}$

$$\lim_{x\to 1}\left(\frac{1}{1+1+\ln x}\right)$$ ← L'Hôpital again.

Answer: $\frac{1}{2}$

Practice

p. 526 #1-19 odd, 27-33 odd

Section 8.7 Indeterminate Forms continued

Indeterminate Powers

Indeterminate Powers

$$\lim_{x\to a}\left[f(x)\right]^{g(x)} \rightarrow \begin{cases} 0^0 \\ \infty^0 \\ 1^\infty \end{cases}$$

Idea :

these three cases can be treated by taking the natural logarithm:

$$y=\left[f(x)\right]^{g(x)} \rightarrow \ln y = g(x)\ln\left[f(x)\right]$$

Example $\lim_{x\to 0+} x^x$

$$\lim_{x\to 0^+} x^x = 0^0$$ Direct Substitution

$$y = \lim_{x\to 0^+} x^x$$ Take natural log of both sides

$$lny = ln \lim_{x\to 0^+} x^x$$

Move log inside the limit

$$lny = \lim_{x\to 0^+} ln\, x^x$$

$$lny = \lim_{x\to 0^+} xln\, x = 0 \cdot (-\infty)$$

$$lny = \lim_{x\to 0^+} \frac{lnx}{\frac{1}{x}} = \frac{-\infty}{\infty}$$ Now use L'H.

$$lny = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} =$$ Simplify

$$lny = \lim_{x\to 0^+} -x = 0$$

$$lny = 0$$ This is not the final answer.

$$e^{lny} = e^0$$

$$y = 1$$

$$y = \lim_{x\to 0^+} x^x = 1$$

Try: $\lim_{x\to 0+}\left(-\ln x\right)^{x} = 1$ $\lim_{x\to\infty}\left(x^{2}+e^{x}\right)^{5/x} = e^{5}$

- p. 526 #21 – 25 all, 49 – 53 odd