Section 9.3

Integral Test and p-series

Integrals are a sum of infinite rectangles under a curve, so they should be related to infinite series.

Integral Test

If f(x) is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then

if $\int_1^\infty f(x)dx$ converges, then $\sum_{n=1}^\infty a_n$ converges

if $\int_1^\infty f(x)dx$ diverges, then $\sum_{n=1}^\infty a_n$ diverges

Example: $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ Converge or Diverge?

Nth term test for Divergence?

Telescopic Series?

Geometric Series?

Is the function positive, continuous and decreasing for $x \geq 1$?

Then try integration.

Example: $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ Converge or Diverge?

Nth term test for Divergence?

Telescopic Series?

Geometric Series?

Is the function positive, continuous and decreasing for $x \geq 1$?

Then try integration.

A series in the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is called a **p-series**.

When p = 1, $\sum_{n=1}^{\infty} \frac{1}{n}$.

This series is called the **harmonic series**.

(In music, the wavelengths of overtones of a vibrating string form a harmonic series.)

We looked at p-integrals earlier this year. What values of p caused the integral to converge?

In p-series,

If $p > 1$, the series converges.

If $p \leq 1$, the series diverges.

This was proven by integration earlier.

Harmonic Series

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \left[\frac{1}{2}\right] + \left[\frac{1}{3} + \frac{1}{4}\right] + \left[\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right] + \left[\frac{1}{9} + \cdots\right] + \cdots$$

$$> 1 + \left[\frac{1}{2}\right] + \left[\frac{1}{4} + \frac{1}{4}\right] + \left[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right] + \left[\frac{1}{16} + \cdots\right] + \cdots$$

$$= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots.$$

Basically this implies that $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$

Examples: Converge or Diverge?

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \qquad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \qquad \sum_{n=1}^{\infty} \frac{1}{3^n}$$

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