Section 9.4

Comparison Tests

$$\sum_{n=0}^{\infty} \frac{1}{2^n}$$ is a geometric series.

$$\sum_{n=0}^{\infty} \frac{n}{2^n}$$ is not

$$\sum_{n=0}^{\infty} \frac{1}{n^2}$$ is a p-series.

$$\sum_{n=0}^{\infty} \frac{1}{n^2+1}$$ is not

$$a_n = \frac{n}{\left(n^2+3\right)^2}$$ is easily integrated.

$$a_n = \frac{n^2}{\left(n^2+3\right)^2}$$ is not

To check the convergence of an unknown series, compare it to a series you know.

Direct Comparison Test (DCT)

For series with positive terms, let $0 \le a_n \le b_n$ for all n

If $\sum b_n$ converges, then $\sum a_n$ (something smaller) converges

If $\sum a_n$ diverges, then $\sum b_n$ (something larger) diverges

Examples

Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$ converge or diverge?

Examples

Does $\sum_{n=1}^{\infty} \frac{5 \ln n}{2n}$ converge or diverge?

Try: $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ $\sum_{n=1}^{\infty} \frac{3^n}{4^n+5}$ $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$

Limit Comparison Test (LCT)

Given two series $\sum a_n$ and $\sum b_n$. Let's compare the terms of each using limits.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$, then $a_n \approx b_n$.

So if b_n converges, a_n also converges. If b_n diverges, a_n also diverges.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, then $a_n \approx cb_n$.

So if b_n converges, a_n also converges. If b_n diverges, a_n also diverges.

The constant doesn't change the convergence of the series since it can be factored out.

Limit Comparison Test (LCT)

Given two series $\sum a_n$ and $\sum b_n$. Let's compare the terms of each using limits.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $a_n < b_n$.

So if b_n converges, the smaller a_n also converges. If b_n diverges, we don't know about a_n .

If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, then $a_n > b_n$.

If b_n diverges, the larger a_n also diverges. If b_n converges, we don't know about a_n.

We are back to the Direct Comparison Test.

Limit Comparison Test (LCT)

For series with positive terms,

If $\lim_{n\to\infty} \frac{a_n}{b_n} =$ **a finite, nonzero constant**, then the series behave the same, both diverge or both converge.

So what about $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$?

More examples: Converge or Diverge.

$$\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$$

More examples: Converge or Diverge.

$$\sum_{n=1}^{\infty} \frac{n^2 - 10}{4n^5 + n^3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

Practice:

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