Section 9.5 Alternating Series

Alternating Series have terms that alternate positive and negative values. (recursive formula will involve -1 raised to a power)

Example: $\sum\_{n=0}^{\infty }\left(-\frac{1}{2}\right)^{n}=\sum\_{n=0}^{\infty }\left(-1\right)^{n}\left(\frac{1}{2}\right)^{n}=1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-… $

**Alternating Series Test**

If an alternating series, $\sum\_{n=1}^{\infty }\left(-1\right)^{n-1}a\_{n}$ satisfies both conditions for all $n\geq 1$, then the alternating series converges.

1. $\lim\_{n\to \infty }a\_{n}=0$ and (2)$ a\_{n+1}\leq a\_{n}$

If condition (2) is not satisfied, the test is inconclusive.

Example: $\frac{2}{1}-\frac{1}{1}+\frac{2}{2}-\frac{1}{2}+\frac{2}{3}-\frac{1}{3}+\frac{2}{4}-\frac{1}{4}+…+\frac{2}{n}-\frac{1}{n} $

Terms are approaching zero (condition 1), but each successive term is not necessarily less than the one before it (condition 2). Therefore, the alternating series test is inconclusive.

Examples: $\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n+1}}{n}$ $\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n}3n}{4n-1}$ $\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n-1}}{n^{2}}$

Alternating Series can be classified as **absolutely convergent** or **conditionally convergent**.

 $\sum\_{}^{}a\_{n}$ is absolutely convergent if $\sum\_{}^{}\left|a\_{n}\right|$ converges.

$\sum\_{}^{}a\_{n}$ is conditionally convergent if $\sum\_{}^{}a\_{n}$ converges, but $\sum\_{}^{}\left|a\_{n}\right|$ diverges.

$\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n+1}}{n}$ converges conditionally because the alternating series converges, but the

non-alternating series $\sum\_{n=1}^{\infty }\frac{1}{n}$ diverges. Does $\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n-1}}{n^{2}}$ converge absolutely or conditionally?

**Estimating Alternating Series**

Estimate $\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n+1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-…$

 0 ½ 1

S1

S2

S3

S4

$$\lim\_{n\to \infty }S\_{n}=$$

The accuracy of our estimation depends on how many terms we include in the partial sum.

**Error** is the difference between the exact value and the estimate. Error is also called the **remainder**.

 Error/Remainder = $R\_{n}=\left|S-S\_{n}\right|$

The maximum value of the error must be less than the next term.

 $R\_{n}<\left|a\_{n+1}\right|$

**Example**: If we use the 4th partial sum to estimate $\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n+1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-…$

 our error must be less than the 5th term.

S4 = a5 = So the actual sum must be

In other words, our sum falls between < S <

**You try**: Given the alternating series $1-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+…+\left(-1\right)^{n+1}\frac{1}{n!} $

Use 6 terms to estimate the sum and find the maximum amount of error in your estimate.