Section 9.6 Ratio and Root Tests

Last 2 tests!!!

Geometric series have a constant ratio between terms. Other series have ratios that are not constant. We will look at the ratio between consecutive terms to determine convergence.

10.7.5 THEOREM (*Ratio Test for Absolute Convergence*). *Let $\sum u_k$ be a series with nonzero terms and suppose that*

$$\rho = \lim_{k \to +\infty} \frac{|u_{k+1}|}{|u_k|}$$

(a) *If $\rho < 1$, then the series $\sum u_k$ converges absolutely and therefore converges.*

(b) *If $\rho > 1$ or if $\rho = +\infty$, then the series $\sum u_k$ diverges.*

(c) *If $\rho = 1$, no conclusion about convergence or absolute convergence can be drawn from this test.*

Use this test for exponential or factorial expressions.

Example

- *Use the ratio test for absolute convergence to determine whether the series converges.*

(a) $$\sum_{k=1}^{\infty}(-1)^k \frac{2^k}{k!}$$

(b) $$\sum_{n=1}^{\infty}\frac{n^2 2^{n+1}}{3^n}$$

Example 5

- *Use the ratio test for absolute convergence to determine whether the series converges.*

(a) $$\sum_{k=1}^{\infty}(-1)^k \frac{2^k}{k!}$$

(b) $$\sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!}{3^k}$$

(a)

$$\rho = \lim_{k\to+\infty}\frac{|u_{k+1}|}{|u_k|} = \lim_{k\to+\infty}\frac{2^{k+1}}{(k+1)!}\cdot\frac{k!}{2^k} = \lim_{k\to+\infty}\frac{2}{k+1} = 0 < 1$$

converges absolutely

Example

- *Use the ratio test for absolute convergence to determine whether the series converges.*

(a) $$\sum_{k=1}^{\infty}(-1)^k \frac{2^k}{k!}$$

(b) $$\sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!}{3^k}$$

(b) $$\rho = \lim_{k\to+\infty}\frac{|u_{k+1}|}{|u_k|} = \lim_{k\to+\infty}\frac{(2k+1)!}{3^{k+1}}\cdot\frac{3^k}{(2k-1)!} = \lim_{k\to+\infty}\frac{(2k+1)(2k)}{3} = +\infty$$

diverges

And finally… the Root Test

- Let $\{a_n\}$ be a sequence and assume that the following limit exists:
  - If $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$ then $\sum_{n=1}^{\infty} a_n$ converges absolutely
  - If $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges
  - If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, the Ratio Test is INCONCLUSIVE

Use this test for expressions raised to the nth power.

Examples

$$\sum_{n=1}^{\infty}\left(\frac{2n+3}{3n+2}\right)^{n}$$

$$\sum_{n=1}^{\infty}\frac{n^{n}}{3^{1+3n}}$$

Try

$$\sum_{n=2}^{\infty} \frac{n^n}{(\ln n)^n}$$

$$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$$