

TEST	SERIES	How it works	Converges	Diverges	Important Information
Geometric Series	$\sum_{n=1}^{\infty} g_1(r)^{n-1}$	Series where each term is generated by multiplying the previous term by the same value (r is that number; called the ratio & it is constant); <u>the power is variable</u> .	$ r < 1$	$ r \geq 1$	<p>Make sure the series is written properly before finding the sum: if $n = 0$, then r^n; if $n = 1$, then r^{n-1}.</p> <p><u>EXAMPLE</u>: $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n} = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n = \sum_{n=1}^{\infty} \frac{1}{4} \left(\frac{1}{4}\right)^{n-1}$</p> <p>Sum can be found with: $S = \frac{g_1}{1-r}$</p>
Harmonic Series	$\sum_{n=1}^{\infty} \frac{1}{n}$	If the numerator is something other than 1, factor it outside the sigma.	NEVER	ALWAYS	<p><u>EXAMPLE</u>: $\sum_{n=1}^{\infty} \frac{5}{n} = 5 \sum_{n=1}^{\infty} \frac{1}{n}$</p> <p>Series is not easy to recognize if not separated into components – use partial fractions for this & you must get subtraction between them. For the type of partial fractions we do, the denominator of your telescoping series <u>must have linear factors</u>.</p> <p>Sum can be found with: $S = \lim_{n \rightarrow \infty} S_n$</p>
Telescoping Series	$\sum_{n=1}^{\infty} (a_n - a_{n+1})$	Series where it's partial sums collapse (cancel each other out). Expand the series and cancel out to simplify to a partial sum: S_n $\lim_{n \rightarrow \infty} S_n$	$\lim_{n \rightarrow \infty} S_n < \infty$	$\lim_{n \rightarrow \infty} S_n = \infty$	<p><u>EXAMPLE</u>: $\sum_{n=1}^{\infty} \frac{1}{2n^2 + 2n} = \sum_{n=1}^{\infty} \frac{1}{2n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{2n} - \frac{1}{2(n+1)}$ (you can factor our a $\frac{1}{2}$ to make it easier to expand)</p> <p>$S_n = \frac{1}{2} - \frac{1}{2n+2}$, so $\lim_{n \rightarrow \infty} S_n = \frac{1}{2}$ therefore $S = \frac{1}{2}$</p> <p>If there is an alternating sign in your p-series, you should use the alternating series test.</p>
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	The power is constant, but the n changes.	$p > 1$	$p \leq 1$	
Nth term Test or Divergence Test	$\sum_{n=1}^{\infty} a_n$	Take $\lim_{n \rightarrow \infty} a_n$.	NOT SURE	As long as the limit $\neq 0$.	For series you don't recognize, use this test first. If it's inconclusive, use the p-series or ratio test next.
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	<u>Check these 3 things</u> : 1) (w/o negative) Is first term > 0 ? 2) Do terms get successively smaller? $a_1 \geq a_2 \geq a_3 \dots$ 3) $\lim_{n \rightarrow \infty} a_n = 0$?	When all 3 conditions are met	If <u>any</u> conditions fail, we are NOT SURE.	Use this test only for alternating series.
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right $; simplify to find r	$ r < 1$	$ r > 1$	If $r = 1$, we have <u>no conclusion</u> . Use this test to create a ratio in order to find radius and/or interval of convergence. Use this test whenever you have exponential or factorial expressions.

TEST	SERIES	How it works ...	Converges	Diverges	Comments
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	Choose a series that you think might match up well with a simplified version of your series. Choose $\frac{1}{n}$ which diverges OR Choose $\frac{1}{n^2}$ which converges. Set up a ratio: $\sum_{n=1}^{\infty} \frac{a_n}{b_n}$ (where b_n is the chosen series) and find the limit L.	If $L > 0$ & the series you choose converges.	If $L > 0$ & the series you choose diverges.	This test is easier than the Direct Comparison Test, but it still requires some skill in choosing the series for comparison. Try thinking of what your series might look like if it could be simplified (mainly look at the leading terms) in order to make a choice for b_n . <u>EXAMPLE:</u> $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2} \right) = \sum_{n=1}^{\infty} \left(\frac{n-1}{n^2} \right)$ Without the -1 in the numerator (which wouldn't matter as you approach infinity), this series simplifies to $\frac{1}{n}$, so that would be good to use for comparison.
Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \sqrt[n]{a_n}$ You take the nth root to get rid of n's in the exponent.	$L < 1$	$L > 1$	If $L = 1$, we have no conclusion. Use this test when you have exponential expressions.
Integral Test	$\sum_{n=1}^{\infty} a_n$	$\int_1^{\infty} f(x) dx$ where $f(x) =$ general term with n being replaced by x .	Series converges if $f(x)$ converges.	Series diverges if $f(x)$ diverges.	This test should only be use when: 1) The series has positive terms. 2) The general expression is one that can be integrated.
Direct Comparison Test	$\sum_{n=1}^{\infty} a_n$	Choose a series (call it series b) whose terms are bigger than your terms. ($a_1 \leq b_1, a_2 \leq b_2, a_3 \leq b_3, \dots$)	If b, who's bigger than a converges, so does a. <i>If b levels off and he's big, then a is certainly going to level off.</i>	If a, who's smaller than b diverges, so does b. <i>If a is already out of control, b is certainly going to be out of control.</i>	Try this test only as a last resort. You have to know what series to choose and how it behaves.