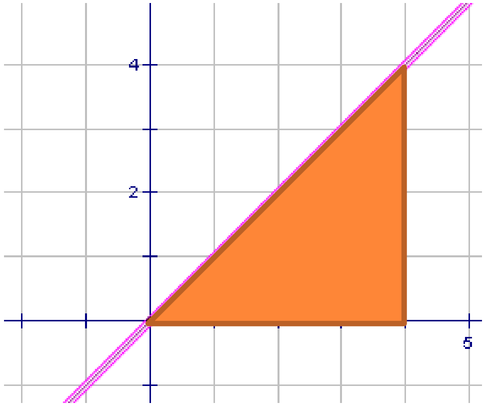
The second part of the Fundamental theorem of calculus

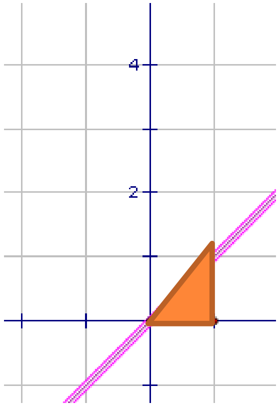
AN INTEGRAL OF A FUNCTION VS. A FUNCTION OF INTEGRALS

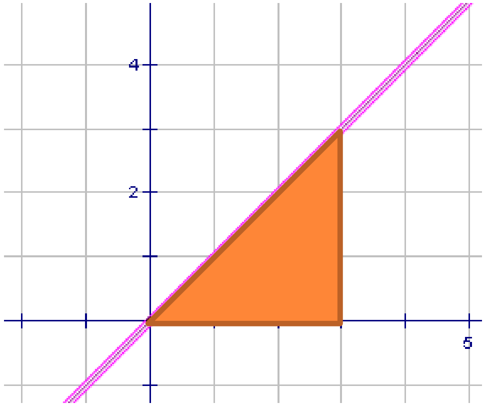
$$\int_0^4 x\,dx$$



$$g(u) = \int_0^u x\,dx$$

$$g(1) = \int_0^1 x\,dx \qquad g(3) = \int_0^3 x\,dx$$





Example

1. An oil rig is spilling oil into the water at the rate of $f(t) = t^3$ barrels/hour.

2. The total oil spilled in 4 hours is given by $\int_0^4 t^3 \, dt$.

3. The total oil spilled in x hours is given by $F(x) = \int_0^x t^3 \, dt$.

4. How can we find the instaneous rate of change in the total oil spilled at time x, $\frac{d}{dx} F(x)$?

Example:

At the 4 hour mark,

what is the rate of change in oil flow?

$$\frac{d}{dx}F(x) = \frac{d}{dx}\int_0^x t^3\, dt$$

What if the lower bound is not zero?

$$\frac{d}{dx}F(x) = \frac{d}{dx}\int_a^x t^3\,dt$$

You Try:

1. $$\frac{d}{dx}\int_0^x \cos t \sin t \, dt = \cos x \sin x$$

2. $$\frac{d}{dx}\int_4^x e^t \sin t \, dt = e^x \sin x$$

3. $$\frac{d}{dx}\int_{-2}^x \frac{\cos t}{4t^2 + t} \, dt = \frac{\cos x}{4x^2 + x}$$

What if upper bound is not just x?

$$\frac{d}{dx}\int_a^{x^2} t^3 dt$$

The Fundamental Theorem of Calculus, Part 2

If *f* is continuous on [a,b], then the function $F(x)=\int_a^x f(t)\,dt$

(where *a* is a constant) has a derivative at every point and

$$\frac{d}{dx}F(x)=\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$$

Combining with the chain rule: $$\frac{d}{dx}\int_a^u f(t)\,dt=f(u)\frac{du}{dx}$$

You try:

1. Evaluate $\frac{d}{dx}\int_0^{x^4} e^t\, dt$

$$\frac{d}{dx}\int_0^{x^4} e^t\, dt = e^{x^4}\left(4x^3\right)$$

2. Evaluate $\frac{d}{dx}\int_5^{3x^2} \text{sint}\, dt$

$$\frac{d}{dx}\int_5^{3x^2} \sin t\, dt = \sin\left(3x^2\right)\left(6x\right)$$

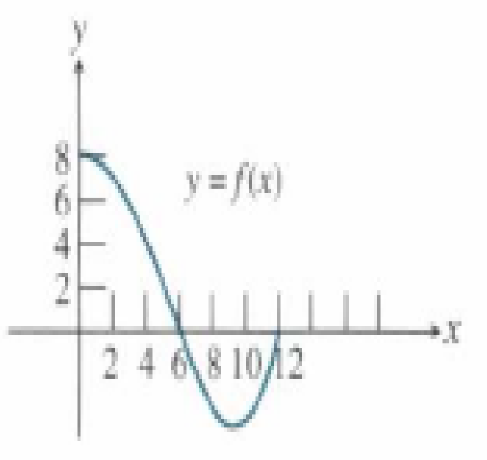
3. Evaluate $\frac{d}{dx}\int_{-4}^{5x^4} \frac{e^t}{4t^2+t}\, dt$

$$\frac{d}{dx}\int_{-4}^{5x^4} \frac{e^t}{4t^2+t}\, dt = \frac{e^{5x^4}}{4\left(5x^4\right)^2+5x^4}\left(20x^3\right)$$

What if both the upper and lower bounds have an x?

$$\frac{d}{dx}\int_{2x}^{x^3} \sin t \, dt$$

53. Let $H(x)=\int_0^x f(t)\,dt$, where f is the continuous function with domain $[0,12]$ graphed here.



(a) Find $H(0)$.

(b) On what interval is H increasing? Explain.

(c) On what interval is the graph of H concave up? Explain.

(d) Is $H(12)$ positive or negative? Explain.

(e) Where does H achieve its maximum value? Explain.

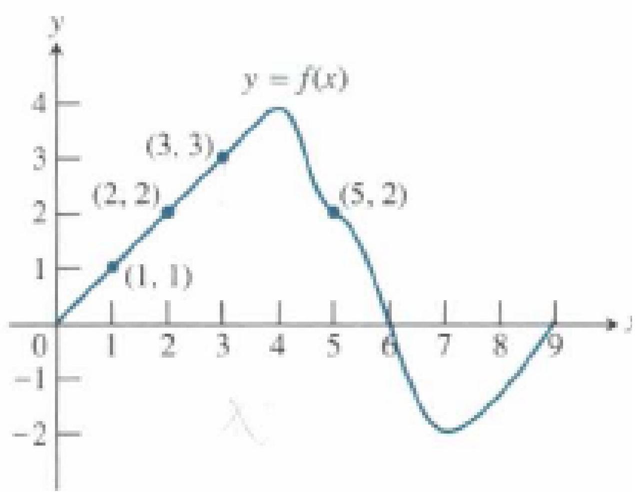
(f) Where does H achieve its minimum value? Explain.

In Exercises 54 and 55, f is the differentiable function whose graph is shown in the figure. The position at time t (sec) of a particle moving along a coordinate axis is

$$s = \int_0^t f(x)\,dx$$

meters. Use the graph to answer the questions. Give reasons for your answers.

54.



(a) What is the particle's velocity at time $t = 5$?

(b) Is the acceleration of the particle at time $t = 5$ positive or negative?

(c) What is the particle's position at time $t = 3$?

(d) At what time during the first 9 sec does s have its largest value?

(e) Approximately when is the acceleration zero?

(f) When is the particle moving toward the origin? away from the origin?

(g) On which side of the origin does the particle lie at time $t = 9$?