Section 8.7 Trapezoidal Approximation

Before we get to new material

1. Estimate the area under the curve $f(x) = 2x^2$ from $x = 0$ to 12 using LRAM, RRAM, and MRAM and three equal-sized partitions.

2. A train is traveling across the country. The following table shows the velocity of the train at 1 hour intervals. Using LRAM and RRAM with 10 partitions and MRAM with 5 partitions, estimate how far the train traveled in 10 hours.

Time (hours)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/hr)	0	12	22	10	5	13	11	6	2	6	1

Estimate the area under the curve $f(x) = 2x^2$ from $x = 0$ to 12 using LRAM, RRAM, and MRAM and three equal-sized partitions.

Three equal partitions means partition size = 4.

The intervals of the three partitions are $[0,4],[4,8],[8,12]$

$LRAM = 4f(0) + 4f(4) + 4f(8)$ Use the left x – value of each interval

$$LRAM = 4(0) + 4(32) + 4(128) = 640$$

$RRAM = 4f(4) + 4f(8) + 4f(12)$ Use the right x – value of each interval

$$RRAM = 4(32) + 4(128) + 4(288) = 1792$$

Estimate the area under the curve $f(x) = 2x^2$

from $x = 0$ to 12 using LRAM, RRAM, and MRAM and

three equal-sized partitions.

The intervals of the three partitions are $[0,4],[4,8],[8,12]$

$MRAM = 4f(2)+4f(6)+4f(10)$ Use the middle $x-$value of each interval

$$MRAM = 4(8)+4(72)+4(200)=1120$$

A train is traveling across the country. The following table shows the velocity of the train at 1 hour intervals. Using LRAM and RRAM with 10 partitions and MRAM with 5 partitions, estimate how far the train traveled in 10 hours.

Time (hours)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/hr)	0	12	22	10	5	13	11	6	2	6	1

LRAM = 0 + 12 + 22 + 10 + 5 + 13 + 11 + 6 + 2 + 6 = 87 miles

RRAM = 12 + 22 + 10 + 5 + 13 + 11 + 6 + 2 + 6 + 1 = 88 miles

MRAM = 2(12) + 2(10) + 2(13) + 2(6) + 2(6) = 94 miles

Unequal Partitions

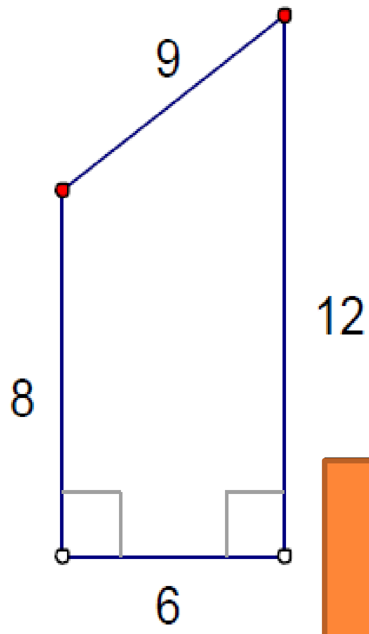
A train is traveling across the country. The following table shows the velocity of the train at various intervals. Using LRAM and RRAM with 5 partitions, estimate how far the train traveled in 8 hours.

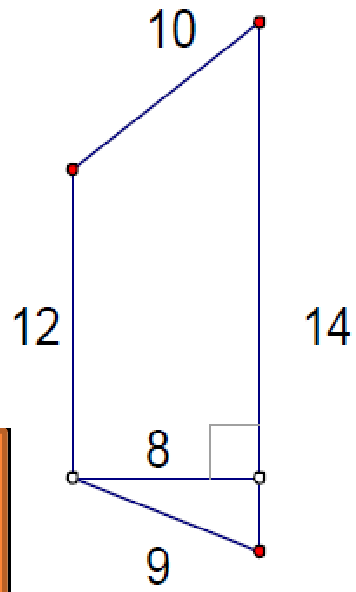
Time (hours)	0	1	3	4	7	8
Velocity (m/hr)	0	12	10	5	6	2

$$LRAM = (1)(0) + (2)(12) + (1)(10) + (3)(5) + (1)(6) = 55 \quad \text{miles}$$

$$RRAM = (1)(12) + (2)(10) + (1)(5) + (3)(6) + (1)(2) = 57 \quad \text{miles}$$

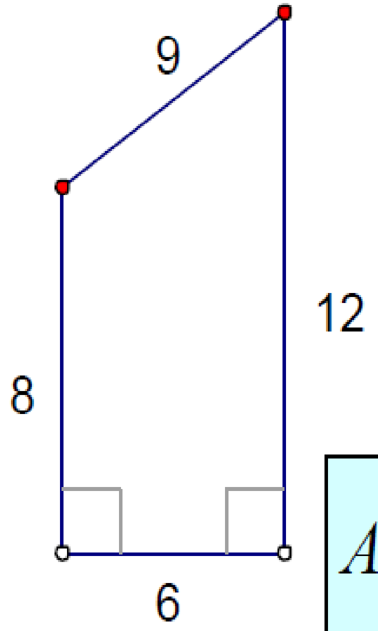
Areas of Trapezoids



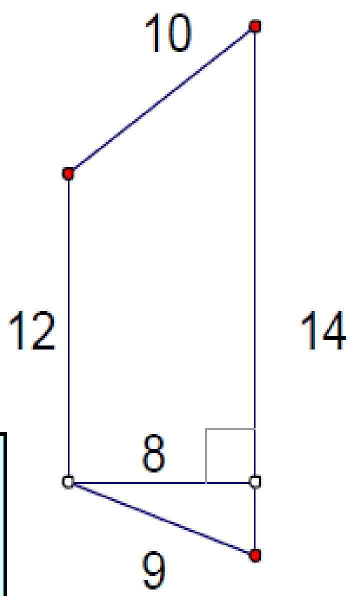


Find the areas of the trapezoids.

Areas of Trapezoids

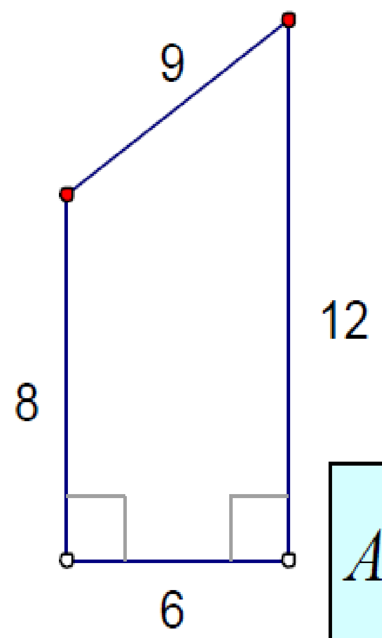


$$A = \frac{1}{2}h\left(b_1 + b_2\right)$$

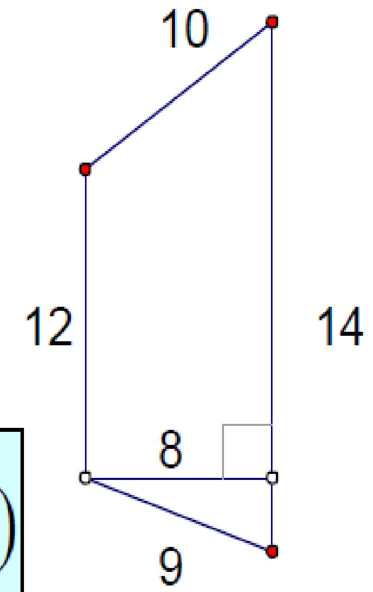


Find the areas of the trapezoids.

Areas of Trapezoids



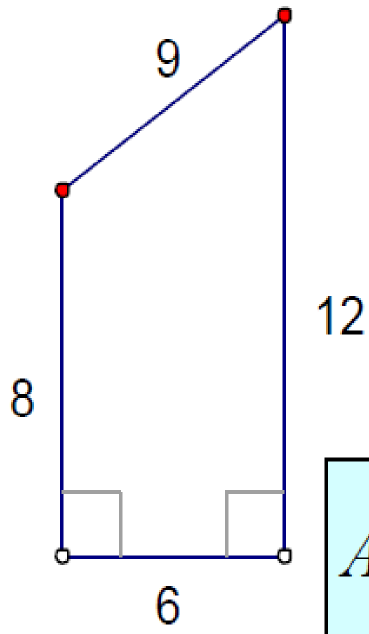
$$A = \frac{1}{2}h\left(b_1 + b_2\right)$$



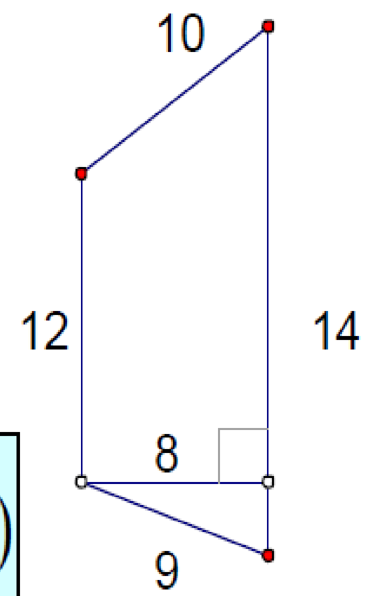
Find the areas of the trapezoids.

$$A = \frac{1}{2}(6)(8+12) = 60$$

Areas of Trapezoids



$$A=\frac{1}{2}h\left(b_1+b_2\right)$$

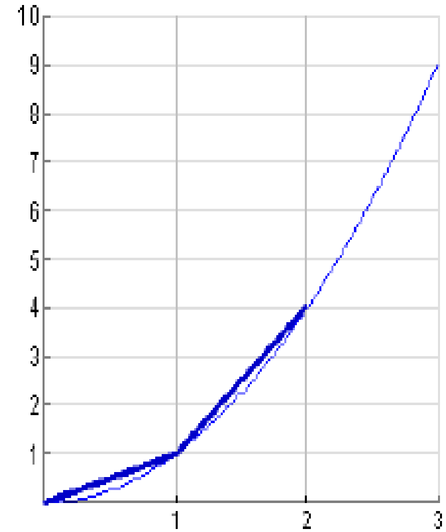


Find the areas of the trapezoids.

$$A=\frac{1}{2}(6)(8+12)=60$$

$$A=\frac{1}{2}(8)(12+14)=104$$

1. Using the function $y = x^2$ from [0,3] to the right, finish drawing in trapezoids by connecting the endpoints of the partitions. Assume there are three partitions.



$y(3) = 9$

$y(2) = 4$

$y(1) = 1$

$y(0) = 0$

2. Looking at these, do you think adding up the trapezoids gives a more accurate or less accurate area estimate than the rectangles did yesterday? WHY? ______________

More accurate, edge of trapezoid more closely matches the curve.

3. Write the formula for area of a trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$A_1 = \frac{1}{2}(1)(0+1) = 0.5$$

$$A_2 = \frac{1}{2}(1)(1+4) = 2.5$$

$$A_2 = \frac{1}{2}(1)(4+9) = 6.5$$

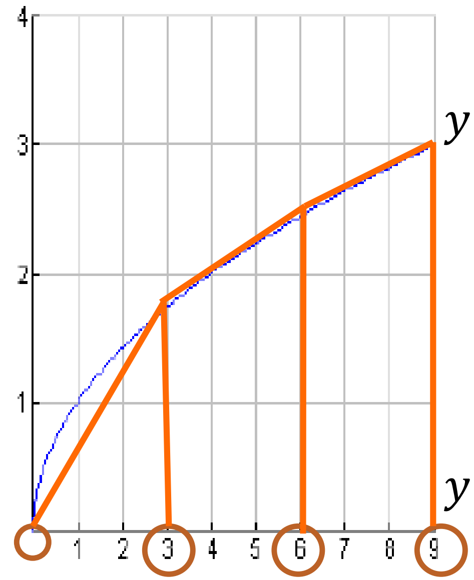
4. Use this formula to estimate the area of the three trapezoids in the above drawing. Add them up to get an estimate for the area under this curve.

$T_3 = 0.5 + 2.5 + 6.5 = 9.5$

Is this estimate an under estimate or over estimate of the actual area between the curve and the x-axis? Justify your answer.

Overestimate, trapezoids are above the curve.

5. Use the same concept to estimate the area under the curve $y = \sqrt{x}$ from [0.9]. USE PARITIONS OF 3



$y(9) = \sqrt{9} = 3$

$y(6) = \sqrt{6}$

$y(3) = \sqrt{3}$

$y(0) = \sqrt{0} = 0$

$$T_3 = \frac{1}{2}(3)\left(0 + \sqrt{3}\right) + \frac{1}{2}(3)\left(\sqrt{3} + \sqrt{6}\right) + \frac{1}{2}(3)\left(\sqrt{6} + 9\right) = 26.0446$$

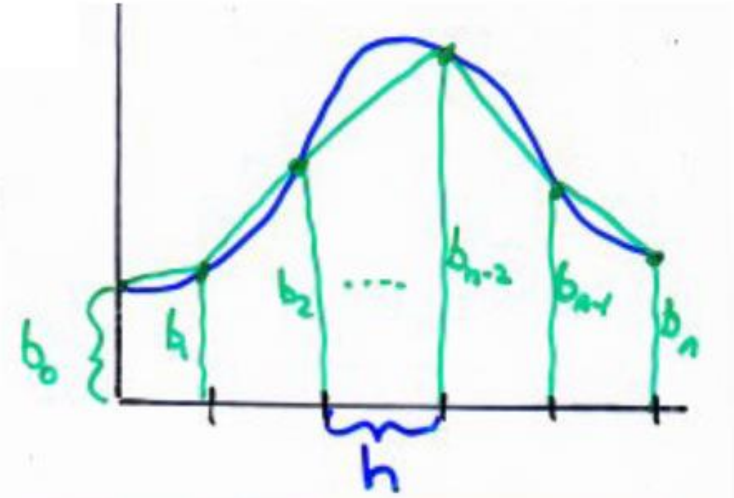
6. Look back at your #4 and #5. Do you notice that in adding the areas up, every partition's height is used twice except the first and the last. **Try to write a general equation for the area under a curve using trapezoids.** Use b_0 for the first base , b_1 for base 1, etc. up to b_n for the last base.

Trapezoidal Rule Summary

$$A = \frac{1}{2} h\left(b_0 + 2b_1 + 2b_2 + \ldots + 2b_{n-1} + b_n\right)$$

$h =$ width of partition

$b =$ value of function at the specific $x -$ value



If partition widths not equal

you must do indiviual trapezoids

$$A = \frac{1}{2} h\left(b_1 + b_2\right)$$

7. Look back at yesterday's notes where we used LRAM and RRAM to estimate these same areas. What relationship does the Trapezoidal Area have with the two of these? Why is that?

$$\text{Trapezoid Approximation} = \frac{\text{LRAM} + \text{RRAM}}{2}$$

8. Coal gas is produced at gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of each month, show the rate at which pollutants are escaping (in tons/month) in the gas:

Time (months)	0	1	2	3	4	5	6
Rate pollutants escape	5	7	8	10	13	16	20

Use trapezoidal rule to estimate the quantity of pollutants that escaped during the first three months. During all six months.

$$T_3 = \frac{1}{2}(1)[5 + 2(7) + 2(8) + (10)] = 22.5\ tons$$

$$T_6 = \frac{1}{2}(1)[5 + 2(7) + 2(8) + 2(10) + 2(13) + 2(16) + 20] = 66.5\ tons$$

A student is driving down Carpenter Upchurch Road in his fancy red Porsche when his radar system warns him of an obstacle 400 feet ahead. He immediately applies the breaks, starts to slow down, and spots a skunk in the road directly in front of him. The “black box” data in the Porsche records the car’s speed every two seconds, producing the following data. The speed decreases throughout the 10 seconds it takes to stop, although not necessarily at a uniform rate.

Time Since Brakes Applied (sec)	Speed (ft/sec)
0	100
2	80
4	50
6	25
8	10
10	0

a) What is your estimate, using trapezoidal rule, of the total distance that the student traveled before coming to rest?

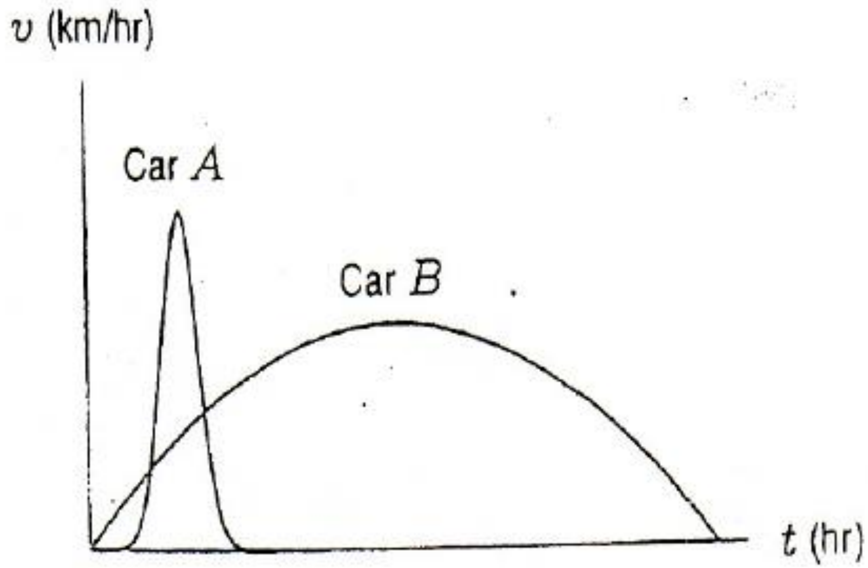
b) Based on this answer, do you think he hit the skunk? (Assume the skunk did not run out the road fast enough!)

$$T_5 = \frac{1}{2}(2)[100 + 2(80) + 2(50) + 2(25) + 2(10) + 0] = 430\ feet$$

We can hope the car passed over him.

10. Two cars start at the same time and travel in the same direction along a straight road. The figure gives the velocity, v, as a function of time, t. Which car:

Attains the larger maximum velocity?
Stops first?
Travels further?



Car A has the largest y-value, so attains the largest maximum velocity.

The velocity of Car A reaches zero first, so it stopped first.

The area under the curve represents the total distance traveled, so Car B travels further.

Find trapezoid approximation using 4 partitions

x	0	3	6	9	12
f(x)	20	30	24	40	52

$$A = \frac{1}{2} h \left(b_0 + 2b_1 + 2b_2 + \ldots + 2b_{n-1} + b_n \right)$$

$$Area = \frac{1}{2}(3)\left(20 + 2 \cdot 30 + 2 \cdot 24 + 2 \cdot 40 + 52\right) \qquad = 390$$

Find trapezoid approximation using 4 partitions

x	0	2	6	9	14
f(x)	20	30	24	40	52

The intervals are not the same width so we have to do each partition separately.

$$Area = \frac{1}{2}h\left(b_1 + b_2\right)$$

$$\begin{aligned} Area &= \frac{1}{2}(2)(20+30)+\frac{1}{2}(4)(30+24)+\frac{1}{2}(3)(24+40)+\frac{1}{2}(5)(40+52) \\ &= 50+108+96+230 \\ &= 484 \end{aligned}$$