Chapter 3Derivatives Review

1. Evaluate.

   a. $\lim_{h\to 0}\frac{\sqrt[4]{16+h}-2}{h}$ $\quad f(x)=\sqrt[4]{x}$

   $f'(16)=$ $\quad f'(x)=\frac{1}{4}x^{-3/4}$

   $\boxed{f'(16)=\frac{1}{32}}$

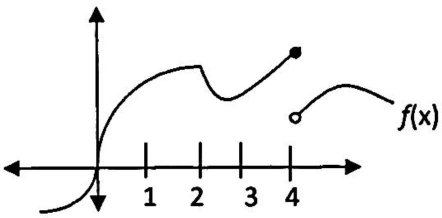
   b. $\lim_{x\to\pi/4}\frac{\tan x-1}{x-\pi/4} = \boxed{f'\left(\frac{\pi}{4}\right)=\left(\frac{2}{\sqrt{2}}\right)^2=2}$

   $f(x)=\tan x$

   $f'(x)=\sec^2 x$

2. True or false.

   a. A function can be continuous at x = 3, but not differentiable at x = 3. T
   b. A function can be differentiable at x = 3, but not continuous at x = 3. F
   c. $f(x)$ is continuous at x = 0 T
   d. $f(x)$ is differentiable at x = 0 F
   e. $f(x)$ is continuous at x = 2 T
   f. $f(x)$ is differentiable at x = 2 F
   g. $f(x)$ is continuous at x = 3 T
   h. $f(x)$ is differentiable at x = 3 T
   i. $f(x)$ is continuous at x = 4 F
   j. $f(x)$ is differentiable at x = 4 F



3. Use the table to answer the following.

X	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	4	2	-6	5
4	1	8	-3	7

$\frac{1}{2}(4)^{-1/2}\cdot 8+\sqrt{4}\cdot 7$

$h'=\frac{1}{2}x^{-1/2}g(x)+\sqrt{x}\cdot g'(x)$

   a. Find $\frac{d}{dx}[g(f(x))]$ when x = 3.

   b. Find $h'(4)$, when $h(x)=\sqrt{x}\cdot g(x)$

$g'(f(3))\cdot f'(3) = -42$

$g'(4)\cdot f'(3) = 7\cdot -6$

16

4. A particle moves along the curve $s(x)=t^3-9t^2+27t$ where s is measured in meters and t in seconds.

   a. Find the acceleration at t = 1 second.

   $-12/\text{sec}^2$

   B. When is the particle at rest?

   $t=3\text{sec}$

5. Determine the values of a and b such that $f(x)$ is differentiable.

$$f(x)=\begin{cases} x^3+b, & x<3 \\ ax^2, & x\geq 3 \end{cases}$$

$3^3+b=a3^2$

$27+b=9a$

$27+b=9\left(\frac{9}{2}\right)$

$b=\frac{81}{2}-27$

$b=\frac{81}{2}-\frac{54}{2}$

$b=\frac{27}{2}$

$3x^2=2ax$

$3(3)^2=2a(3)$

$27=6a$

$\frac{9}{2}=\frac{27}{6}=a$

P215 Review

EXERCISES

1. Estimating the slopes of the tangent lines at $x = 2, 3,$ and 5, we obtain approximate values 0.4, 2, and 0.1. The slope of the tangent line at $x = 7$ is negative, so $f'(7) < 0$. Arranging the numbers in increasing order, we have: $f'(7) < 0 < f'(5) < f'(2) < 1 < f'(3)$.

2. $2^6 = 64$, so $f(x) = x^6$ and $a = 2$.

9. $B'(1990)$ is the rate at which the total value of U.S. banknotes in circulation is changing in billions of dollars per year. To estimate the value of $B'(1990)$, we will average the difference quotients obtained using the times $t = 1985$ and $t = 1995$. Let $A = \dfrac{B(1985) - B(1990)}{1985 - 1990} = \dfrac{182.0 - 268.2}{-5} = 17.24$ and

$C = \dfrac{B(1995) - B(1990)}{1995 - 1990} = \dfrac{401.5 - 268.2}{5} = 26.66$. Then

$$B'(1990) = \lim_{t \to 1990} \frac{B(t) - B(1990)}{t - 1990} \approx \frac{A + C}{2} = \frac{17.24 + 26.66}{2} = 21.95 \text{ billions of dollars/year.}$$

13. $y = (x^4 - 3x^2 + 5)^3 \quad \Rightarrow$

$$y' = 3(x^4 - 3x^2 + 5)^2 \frac{d}{dx}(x^4 - 3x^2 + 5) = 3(x^4 - 3x^2 + 5)^2(4x^3 - 6x) = 6x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$$

15. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3} \quad \Rightarrow \quad y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3} = \dfrac{1}{2\sqrt{x}} - \dfrac{4}{3\sqrt[3]{x^7}}$

17. $y = 2x\sqrt{x^2 + 1} \quad \Rightarrow$

$$y' = 2x \cdot \tfrac{1}{2}(x^2 + 1)^{-1/2}(2x) + \sqrt{x^2 + 1}\,(2) = \frac{2x^2}{\sqrt{x^2 + 1}} + 2\sqrt{x^2 + 1} = \frac{2x^2 + 2(x^2 + 1)}{\sqrt{x^2 + 1}} = \frac{2(2x^2 + 1)}{\sqrt{x^2 + 1}}$$

19. $y = \dfrac{t}{1 - t^2} \quad \Rightarrow \quad y' = \dfrac{(1 - t^2)(1) - t(-2t)}{(1 - t^2)^2} = \dfrac{1 - t^2 + 2t^2}{(1 - t^2)^2} = \dfrac{t^2 + 1}{(1 - t^2)^2}$

21. $y = \tan\sqrt{1 - x} \quad \Rightarrow \quad y' = \left(\sec^2\sqrt{1 - x}\right)\left(\dfrac{1}{2\sqrt{1 - x}}\right)(-1) = -\dfrac{\sec^2\sqrt{1 - x}}{2\sqrt{1 - x}}$

25. $y = \dfrac{\sec 2\theta}{1 + \tan 2\theta} \quad \Rightarrow$

$$y' = \frac{(1 + \tan 2\theta)(\sec 2\theta \tan 2\theta \cdot 2) - (\sec 2\theta)(\sec^2 2\theta \cdot 2)}{(1 + \tan 2\theta)^2} = \frac{2\sec 2\theta\,[(1 + \tan 2\theta)\tan 2\theta - \sec^2 2\theta]}{(1 + \tan 2\theta)^2}$$

$$= \frac{2\sec 2\theta\,(\tan 2\theta + \tan^2 2\theta - \sec^2 2\theta)}{(1 + \tan 2\theta)^2} = \frac{2\sec 2\theta\,(\tan 2\theta - 1)}{(1 + \tan 2\theta)^2} \quad [1 + \tan^2 x = \sec^2 x]$$

27. $y = (1 - x^{-1})^{-1} \quad \Rightarrow$

$$y' = -1(1 - x^{-1})^{-2}[-(-1x^{-2})] = -(1 - 1/x)^{-2}x^{-2} = -((x - 1)/x)^{-2}x^{-2} = -(x - 1)^{-2}$$

31. $y = \cot(3x^2 + 5) \quad \Rightarrow \quad y' = -\csc^2(3x^2 + 5)(6x) = -6x\csc^2(3x^2 + 5)$

33. $y = \sin(\tan\sqrt{1 + x^3}) \quad \Rightarrow \quad y' = \cos(\tan\sqrt{1 + x^3})(\sec^2\sqrt{1 + x^3})[3x^2/(2\sqrt{1 + x^3})]$

35. $y = \tan^2(\sin\theta) = [\tan(\sin\theta)]^2 \quad \Rightarrow \quad y' = 2[\tan(\sin\theta)] \cdot \sec^2(\sin\theta) \cdot \cos\theta$

37. $y = (x\tan x)^{1/5} \;\Rightarrow\; y' = \frac{1}{5}(x\tan x)^{-4/5}(\tan x + x\sec^2 x)$

39. $f(t) = \sqrt{4t+1} \;\Rightarrow\; f'(t) = \frac{1}{2}(4t+1)^{-1/2}\cdot 4 = 2(4t+1)^{-1/2} \;\Rightarrow$
$f''(t) = 2(-\frac{1}{2})(4t+1)^{-3/2}\cdot 4 = -4/(4t+1)^{3/2}$, so $f''(2) = -4/9^{3/2} = -\frac{4}{27}$.

45. $y = 4\sin^2 x \;\Rightarrow\; y' = 4\cdot 2\sin x\cos x$. At $\left(\frac{\pi}{6}, 1\right)$, $y' = 8\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$, so an equation of the tangent line is $y - 1 = 2\sqrt{3}\left(x - \frac{\pi}{6}\right)$, or $y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$.

47. $y = \sqrt{1+4\sin x} \;\Rightarrow\; y' = \frac{1}{2}(1+4\sin x)^{-1/2}\cdot 4\cos x = \dfrac{2\cos x}{\sqrt{1+4\sin x}}$. At $(0,1)$, $y' = \dfrac{2}{\sqrt{1}} = 2$, so an equation of the tangent line is $y - 1 = 2(x-0)$, or $y = 2x+1$.

51. $y = \sin x + \cos x \;\Rightarrow\; y' = \cos x - \sin x = 0 \;\Leftrightarrow\; \cos x = \sin x$ and $0 \le x \le 2\pi \;\Leftrightarrow\; x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$, so the points are $\left(\frac{\pi}{4}, \sqrt{2}\right)$ and $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$.

69. f is not differentiable: at $x = -4$ because f is not continuous, at $x = -1$ because f has a corner, at $x = 2$ because f is not continuous, and at $x = 5$ because f has a vertical tangent.

84. $\displaystyle\lim_{x\to 1}\frac{x^{17}-1}{x-1} = \left[\frac{d}{dx}x^{17}\right]_{x=1} = 17(1)^{16} = 17$

85. $\displaystyle\lim_{h\to 0}\frac{\sqrt[4]{16+h}-2}{h} = \left[\frac{d}{dx}\sqrt[4]{x}\right]_{x=16} = \frac{1}{4}x^{-3/4}\Big|_{x=16} = \frac{1}{4\left(\sqrt[4]{16}\right)^3} = \frac{1}{32}$

86. $\displaystyle\lim_{\theta\to\pi/3}\frac{\cos\theta - 0.5}{\theta - \pi/3} = \left[\frac{d}{d\theta}\cos\theta\right]_{\theta=\pi/3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$