AB Calculus
No Calculator

INTEGRATION REVIEW #2
U-Substitution

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|---|---|
| 1. $\frac{1}{2}\int e^{2x+5}dx = \frac{1}{2}e^{2x+5} + C$ | 11. $\frac{1}{\pi}\int \sin(\pi x)dx = -\frac{1}{\pi}\cos(\pi x) + C$ |
| 2. $\frac{1}{2}\int \frac{2x}{x^2+4}dx = \frac{1}{2}\ln\lvert x^2+4\rvert + C$ | 12. $\int(\sec^3 x \tan x)dx$ $= \frac{1}{\cos^3}\cdot\frac{\sin}{\cos} = \frac{-u^{-3}}{-3} = \boxed{\frac{1}{3}\cos^{-3}x + C}$; $\sec^2 x\, \sec x \tan x$; $\frac{1}{3}u^3 = \boxed{\frac{\sec^3 x}{3} + C}$ |
| 3. $\frac{1}{6}\int \frac{6x^2}{\sqrt{2x^3+5}}dx = \frac{1}{6}(2x^3+5)^{1/2}\cdot 2 + C$ | 13. $\frac{2}{3}\int \frac{3}{2}\sqrt{x}\sin(1+x^{3/2})dx = -\frac{2}{3}\cos(1+x^{3/2}) + C$ |
| 4. $\int(2\sin x\cos^2 x)dx = -\frac{2u^3}{3} = -\frac{2}{3}\cos^3 x + C$ | 14. $\int(\sqrt{\cot x}\csc^2 x)dx = -\frac{2}{3}(\cot x)^{3/2} + C$ |
| 5. $\int(e^x\cos e^x)dx = \sin e^x + C$ | 15. $\int \sin x \sec^2(\cos x)dx = -\tan(\cos x) + C$ |
| 6. $\int(e^{\sin x}\cos x)dx = e^{\sin x} + C$ | 16. $\int\frac{dx}{x\ln x} = \int\frac{1}{x}\cdot\frac{1}{(\ln x)}dx = \ln(\ln x) + C$ |
| 7. $\int\frac{1-\sin x}{x+\cos x}dx = \ln\lvert x+\cos x\rvert + C$ | 17. $\int \cot x\, dx = \int\frac{\cos x}{\sin x}dx = \ln\lvert\sin x\rvert + C$ |
| 8. $-\frac{1}{3}\int\frac{-3}{4-3x}dx = \frac{1}{-3}\ln\lvert 4-3x\rvert + C$ | 18. $\int\frac{e^{1/x}}{x^2}dx = -\int e^{x^{-1}}\cdot -x^{-2}dx = -e^{x^{-1}} + C$ |
| 9. $\int(e^x\sqrt{1+4e^x})dx = \frac{1}{4}(1+4e^x)^{3/2}\cdot\frac{2}{3} + C$ | 19. $\int\frac{\tan^{-1}x}{1+x^2}dx = \frac{1}{2}u^2 = \frac{1}{2}(\tan^{-1}x)^2 + C$ |
| 10. $\int(x^2\sqrt{1+x})dx = \int(u-1)^2\sqrt{u}\,du$ | 20. $\int(x^5)(\sqrt[3]{x^3+1})dx = \frac{1}{3}\int 3x^2 x^3 (x^3+1)^{1/3}dx$ |

$u = 1+x$
$u-1 = x$
$du = 1\,dx$

$= \int(u^2-2u+1)\sqrt{u}\,du$

$= \int(u^{5/2}-2u^{3/2}+u^{1/2})\,du$

$\frac{2u^{7/2}}{7} - \frac{2\cdot 2u^{5/2}}{5} + \frac{2u^{3/2}}{3} + C$

$\frac{2(1+x)^{7/2}}{7} - \frac{4(1+x)^{5/2}}{5} + \frac{2(1+x)^{3/2}}{3} + C$

$u = x^3+1$
$du = 3x^2dx$
$x^3 = u-1$

$\frac{1}{3}\int(u-1)u^{1/3}\,du$

$\frac{1}{3}\int u^{4/3}-u^{1/3}\,du$

$\frac{1}{3}\left(\frac{3u^{7/3}}{7} - \frac{3u^{4/3}}{4}\right) + C$

$\frac{(x^3+1)^{7/3}}{7} - \frac{(x^3+1)^{4/3}}{4} + C$