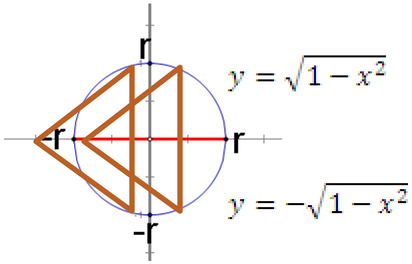
# Volume by Known Cross Sections

- A solid can be created with cross sections of any shape.
- Create a solid whose base is a circle with radius 1 and whose parallel cross sections are equilateral triangles perpendicular to the x-axis.

To find volume of the solid:

Step 1: Draw base and cross section.

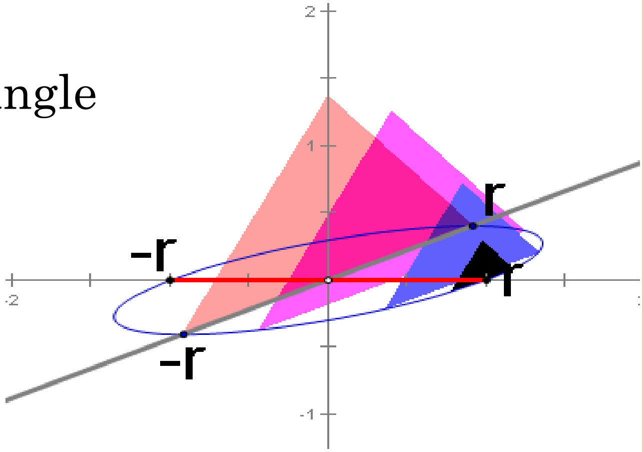



Step 2: Write an expression for the edge of the cross section.
Top curve – bottom curve

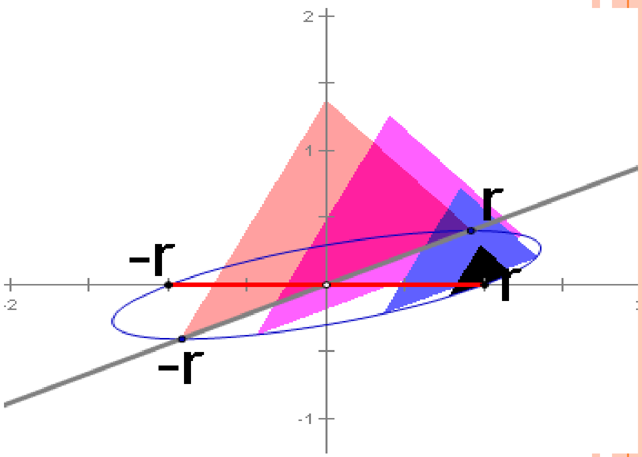
$x^2 + y^2 = 1$ $\quad y = \pm\sqrt{1-x^2}$

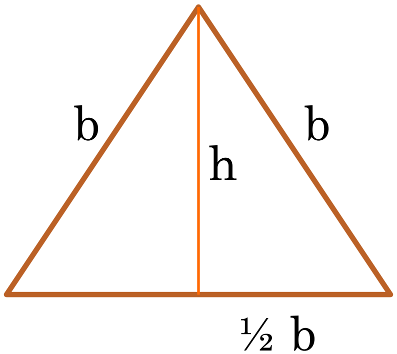
$$edge = \sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right) = 2\sqrt{1-x^2}$$

Step 3: Write an expression for the area of the triangle in terms of the edge touching the base.



Area of a triangle = ½ bh$= \frac{1}{2} b \cdot \frac{\sqrt{3}b}{2} = \frac{\sqrt{3}b^2}{4}$




$$h^2 + \left(\frac{1}{2}b\right)^2 = b^2$$

$$h^2 = \frac{3}{4}b^2$$

$$h = \frac{\sqrt{3}b}{2}$$

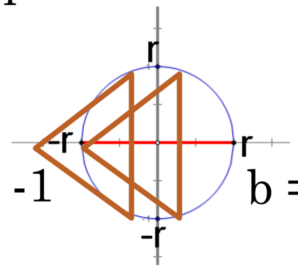
Step 3: Write an expression for the area of the triangle in terms of the edge touching the base.

$$edge = 2\sqrt{1 - x^2}$$

$$\text{Area} = \frac{\sqrt{3}edge^2}{4} = \frac{\sqrt{3}\left(2\sqrt{1 - x^2}\right)^2}{4} = \frac{\sqrt{3} \cdot 4(1 - x^2)}{4} = \sqrt{3}(1 - x^2)$$

Step 4: Write an expression for volume by adding up infinite number of triangles from a to b

$$Volume = \int_{-1}^{1} \sqrt{3}(1 - x^2)dx = \sqrt{3}\int_{-1}^{1} (1 - x^2)dx$$



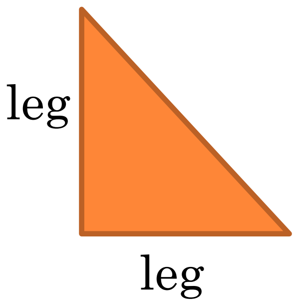

a= -1 b = 1

Step 5: Evaluate integral.

Area = $\sqrt{3}(1 - x^2)$

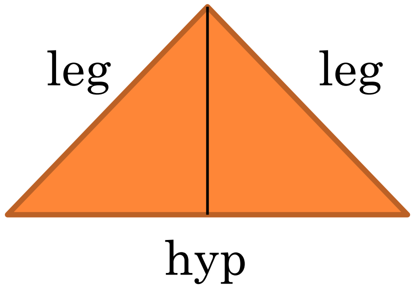
Volume = $\frac{4}{3}\sqrt{3}$ cubic units = 2.309 $u^3$

# Area formulas for Isosceles Right Triangles




If leg is touching the base:

$\text{Area} = \frac{1}{2}\, bh = \frac{1}{2}\, \text{leg} \times \text{leg} = \frac{1}{2}\, \text{leg}^2$

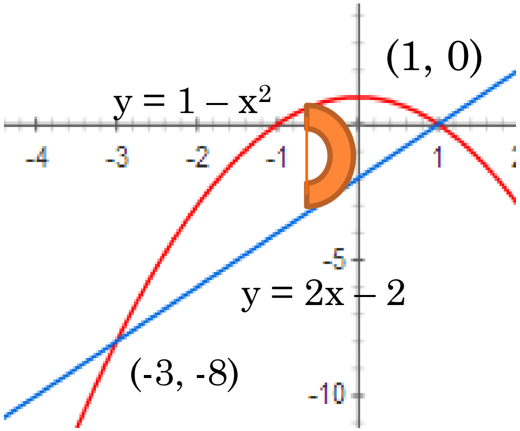



If hypotenuse is touching the base:

$\text{Area} = \frac{1}{2}\, bh = \frac{1}{2}\, \text{hyp} \times \frac{1}{2}\, \text{hyp} = \frac{hyp^2}{4}$

Height = ½ hyp

# Second Example: Find the volume of a solid whose base is bound by $y = 2x - 2$ and $y = 1 - x^2$ with semicircle cross sections perpendicular to the x-axis.




1. Draw base and cross sections
2. Edge touching the base = Top – bottom

   $1 - x^2 - (2x - 2) = 3 - x^2 - 2x$ = diameter

3. Area = $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{d}{2}\right)^2 = \frac{1}{2}\pi\left(\frac{3 - x^2 - 2x}{2}\right)^2 = \frac{1}{8}\pi(3 - x^2 - 2x)^2$

Intersection points

$2x - 2 = 1 - x^2$

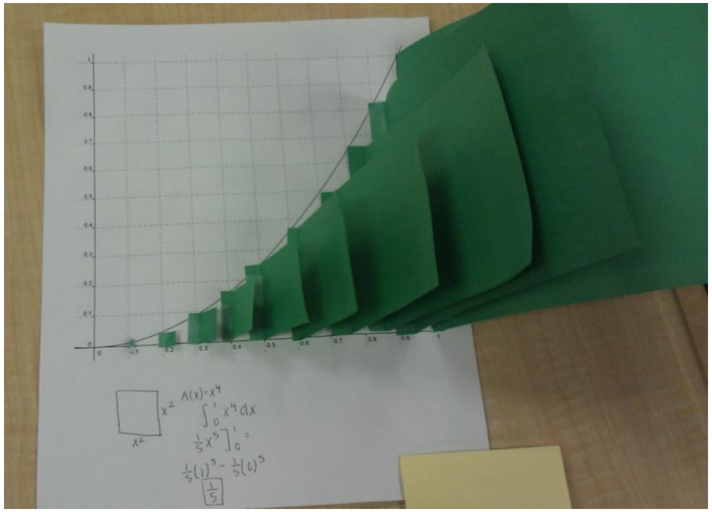
$x^2 + 2x - 3 = 0$

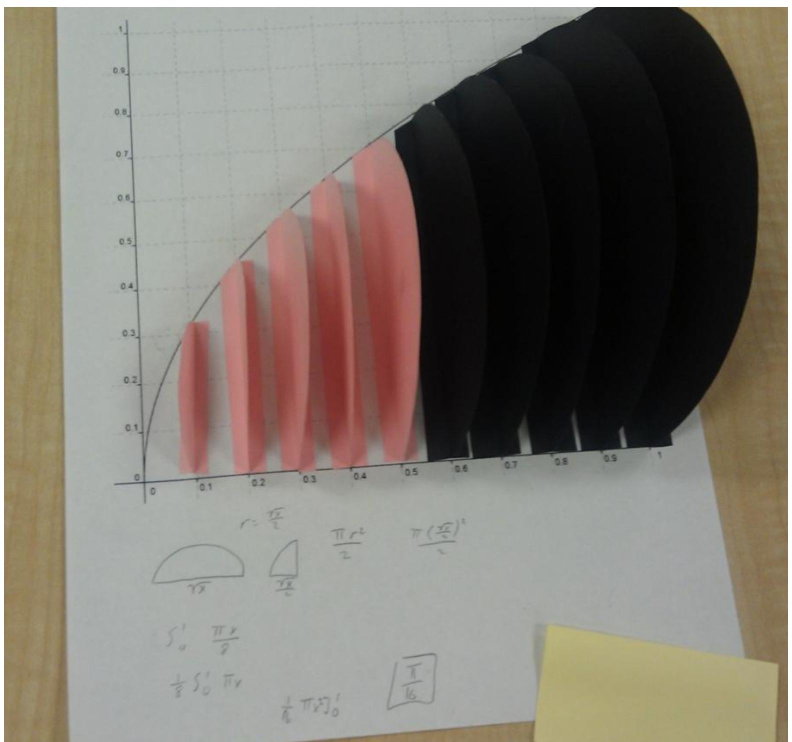
$(x - 1)(x + 3) = 0$

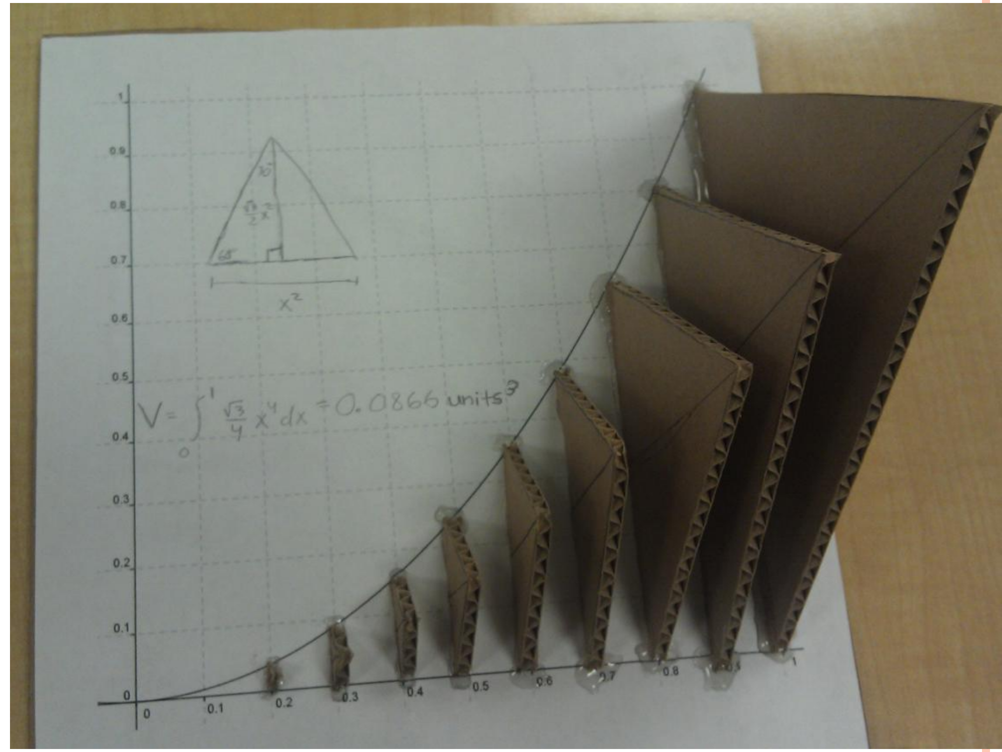
$x = 1, -3$

4. Volume = $\int_{-3}^{1} \frac{1}{8}\pi(3 - x^2 - 2x)^2\, dx = \frac{1}{8}\pi \int_{-3}^{1}(3 - x^2 - 2x)^2 dx$

5. Volume = $\frac{\pi}{8}(34.1333) = 4.2666\pi = 13.404\, u^3$







Build solid in groups by attaching 10 cross sections to the base region.

Estimate the volume by adding 10 prisms whose height is the space between the cross sections.
Then calculate the exact volume of your solid.