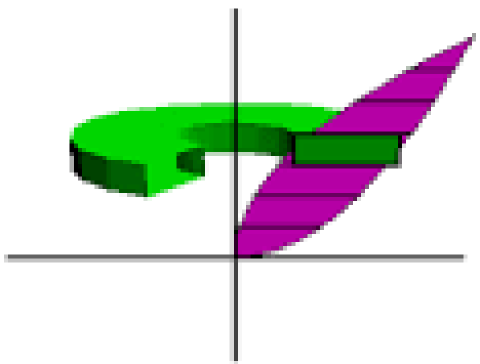
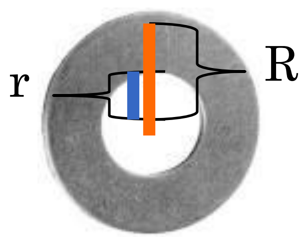
Volume with Washers

- What happens when the region we are revolving is not adjacent to the axis of revolution?



We get a hole!

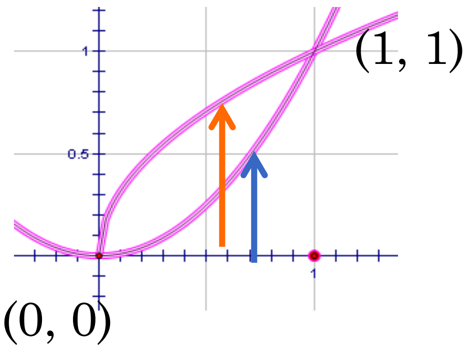
The volume of the solid will be the sum of washers (a circle with a hole).



How do you find the area of a washer?

Area = outer circle – inner circle = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

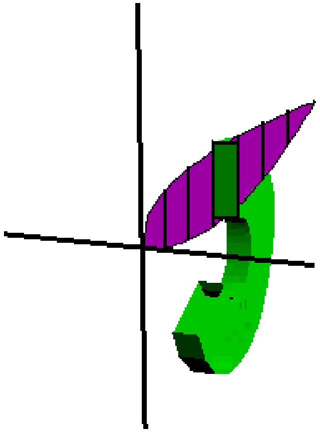
Find the volume of the region bound by $y = x^2$ and $y = \sqrt{x}$ rotated about the the x-axis.

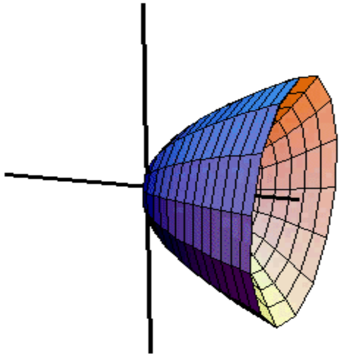


Find intersection points

$$\text{Volume} = \int_0^1 \pi R^2 - \pi r^2 dx$$

Outer $R = \sqrt{x}$ Inner $r = x^2$





$$Volume = \int_0^1 \pi(\sqrt{x})^2 - \pi(x^2)^2 dx$$

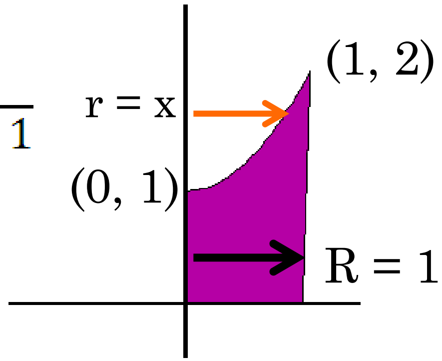
$V = 0.3\pi$

Washer demo

Find the volume of the region bound by $x = 0$, $y = 0$, $x = 1$ and $y = x^2 + 1$ revolved about the y-axis.

$y = x^2 + 1$

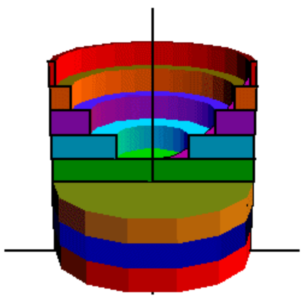
$r = x = \sqrt{y - 1}$

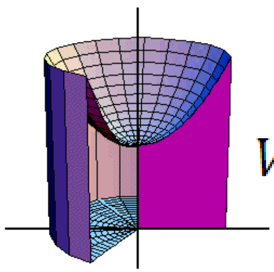


The volume is the sum of disks and the sum of washers from y-value to y-value.

$$Volume = \int_0^1 \pi R^2 dy + \int_1^2 \pi R^2 - \pi r^2 dy$$

Disks + Washers



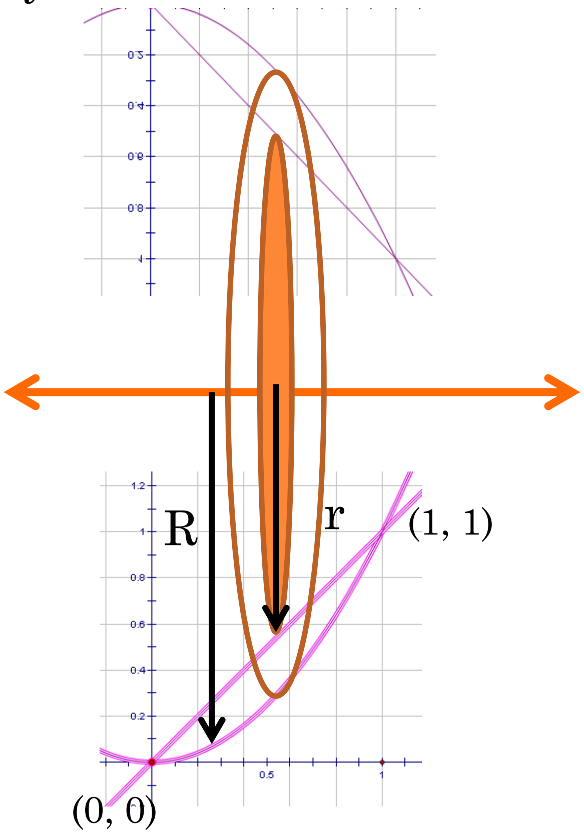


$$Volume = \int_0^1 \pi 1^2 dy + \int_1^2 \pi 1^2 - \pi(\sqrt{y-1})^2 dy$$

Volume = 1.5π

Washer demo

Find the volume of the region bound by $y = x$ and $y = x^2$ revolved about the line $y = 3$



Volume is sum of washers from x-value to x-value

$$Volume = \int_0^1 \pi R^2 - \pi r^2 dx$$

$$Volume = \int_0^1 \pi(Top - Bottom)^2 - \pi(top - bottom)^2 dx$$

$R = 3 - x^2$ $\qquad$ $r = 3 - x$

$$Volume = \int_0^1 \pi(3 - x^2)^2 - \pi(3 - x)^2 dx$$

$$= \frac{13}{15}\pi \ units^3$$

- Practice p391 # 8-11, 13, 14, 16 – 18
- Calculaugh 57