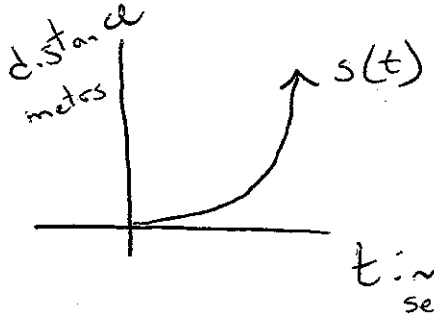
Section 3.8 Higher Order Derivatives

Position, Velocity and Acceleration



This is the position function. It shows displacement (distance from starting point) as a function of time.

$$\text{Velocity} = \frac{\text{change in position}}{\text{change in time}}$$

- shows both speed and direction
  pos velocity means forward/upwards
  neg velocity means backwards/down
  speed = $|\text{velocity}|$

Average velocity is the slope between two positions.
Instantaneous velocity is velocity at a specific time or derivative at a point. $v(t) = s'(t)$ m/s

Acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$

- derivative of velocity $a(t) = v'(t) = s''(t)$ m/sec^2

Jerk = $\frac{\text{change in acceleration}}{\text{change in time}}$

$j(t) = a'(t) = v''(t) = s'''(t)$ m/sec^3

4th derivative = $s^{(4)}(t)$

Example 1: The position of a particle is given by $s(t) = t^3 - 6t^2 + 9t$, with time in seconds and position in meters.

a) Find acceleration after 4 seconds.

b) When is the particle not moving?

a) $s(t) = t^3 - 6t^2 + 9t$

$v(t) = s'(t) = 3t^2 - 12t + 9$

$a(t) = v'(t) = 6t - 12$

$a(4) = 6(4) - 12 = 24 - 12 = 12 \text{ m/sec}^2$

b) $v(t) = 3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t - 1)(t - 3) = 0$

$t = 1$ or 3

Not moving at 1 sec and 3 sec.

Example 2: On the moon, an arrow is shot upwards at a velocity of 58 m/sec. Its height after t seconds is given by $h(t) = 58t - 0.83t^2$.
What is its velocity when it hits the ground?

$h(t) = 58t - 0.83t^2$

$v(t) = h'(t) = 58 - 1.66t$

$v\left(\frac{58}{.83}\right) = 58 - 1.66\left(\frac{58}{.83}\right)$

$= -58$ m/sec

$h(t) = 58t - .83t^2 = 0$

$t(58 - .83t) = 0$

$t = 0 \quad 58 - .83t = 0$

$58 = .83t$

$\frac{58}{.83} = t$