AP Exam Free Response Practice for Power Series

The function $f$ is defined by the power series

$$f\left(x\right)=\left(x-1\right)-\frac{\left(x-1\right)^{2}}{2}+\frac{\left(x-1\right)^{3}}{3}-\frac{\left(x-1\right)^{4}}{4}+…=\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n-1}\left(x-1\right)^{n}}{n}$$

For all real numbers x for which the series converges.

1. Determine the interval of convergence for $f$. Justify your answer.
2. Given $g\left(x\right)=f'(x)$, find the first three terms and the general term of the power series for $g$.
3. Find a rational function that is identical to $g$ over its interval of convergence.
4. Let $h$ be the function defined by $h\left(x\right)=f\left(x^{3}+1\right).$ Find a function that represents $h’$.