THE CHAIN RULE

Section 2.4

Estimating derivatives of a function defined by a table of values

Example: Let D(t) be the U.S. National debt at time t. D(t) is measured in billions of dollars.

Estimate and interpret D'(1990)

t	D(t)
1980	930.2
1985	1945.9
1990	3233.3
1995	4974.0
2000	5674.2

D’(1990) can be calculated two ways.

1. “straddle the point”

Calculate the rate of change from 1985 to 1995.

t	D(t)
1980	930.2
1985	1945.9
1990	3233.3
1995	4974.0
2000	5674.2

D'(1990) can be calculated two ways.

2. Calculate the rate of change on both sides, then average the two rates.

1985 to 1990

1990 to 1995

t	D(t)
1980	930.2
1985	1945.9
1990	3233.3
1995	4974.0
2000	5674.2

D'(1990) = approx 303 billion dollars per yr. This is the rate the debt was increasing in 1990.

Problem: Find the derivative of

$$f(x) = (2x^2 - 4x + 1)^{60}$$

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

$$f(x) = (2x^2 - 4x + 1)^{60}$$

$$f(x) = [g(x)]^{60}$$

$$g(x) = 2x^2 - 4x + 1$$

$f'(x) = 60($[redacted]$)^{59} \cdot$

Example: $f(x) = \sqrt{x^2 + 1} = (x^2 + 1)^{1/2}$

Outside

$(\quad)^{1/2}$ $\qquad$ $f'(x) = \frac{1}{2}(\quad)^{-1/2} \cdot$

Example: $f(x) = \sin^2 x = [\sin x]^2$

Outside

$(\quad)^2$ $\qquad$ $f'(x) = 2(\quad)^1 \cdot$

You Try:

$y = (3x - 4)^{10}$

$y' = 10(3x - 4)^9 \cdot 3$

$y' = 30(3x - 4)^9$

$y = \sin(7 - 5x)$

$y' = \cos(7 - 5x) \cdot -5$

$y' = -5\cos(7 - 5x)$

Chain Rule with other rules

$y = (2x + 5)^5 \cdot (x^3 - x + 1)^4$

$y = \left(\frac{x-2}{2x+1}\right)^9$

Using Chain Rule Multiple Times

Ex. $y = (1 + \cos(2x))^2$

Ex. $y = \tan[5 - \sin(2x)]$

Ex. $y = \sqrt{\sec(x^3)}$