Chapter 3 Derivatives

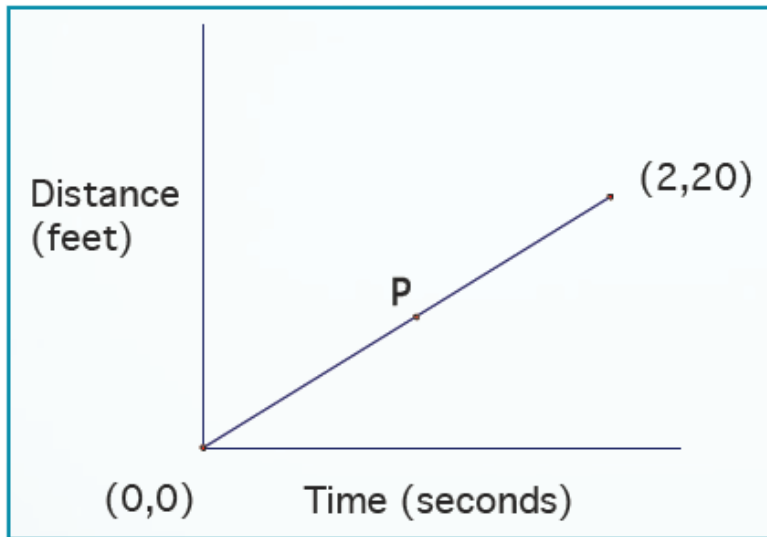**Distance (feet)** (vertical axis)
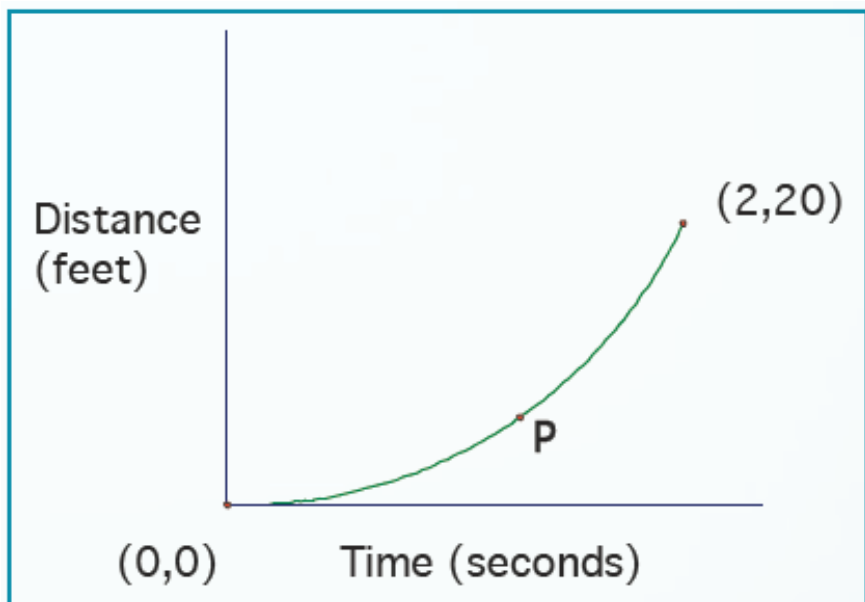
(2,20)

P

(0,0) — Time (seconds) (horizontal axis)

What is the slope of the line?

What is the average rate of change from t=0 to t=2?

What is the slope at point P?

What is the average rate of change at point P?

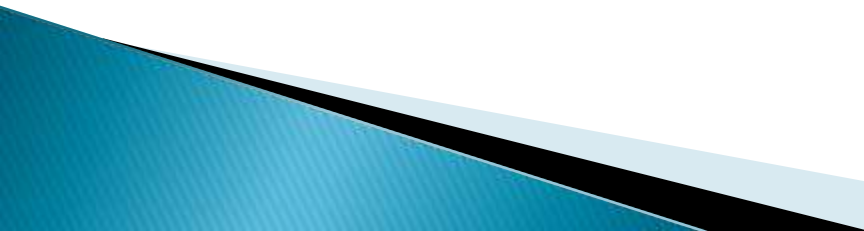Distance (feet) vs Time (seconds). Points (0,0), P, and (2,20) shown on an upward-curving graph.

What is the average rate of change from t=0 to t=2?

What is the average slope?

What is the slope at point P?

What is the rate of change at point P?

Definitions

- **<u>Average rate of change</u>**: slope between two points

- **<u>Instantaneous rate of change</u>**: slope or rate of change at a specific point, also called the <u>derivative at a point</u>

- **<u>Derivative</u>**: equation used to calculate slopes/derivatives at specific points on a function

- For linear graphs, average and instantaneous rates of change are the same
- For all other graphs, average and instantaneous rates are not the same

- If you travel 100 miles in 2 hours, your average rate of change is 50 mph, but your instantaneous rate of change varies from moment to moment during the 2 hour period.

History Note p139

Notation

Given function $f(x)$

$f'(x)$ = derivative of function f

$f'(3)$ = derivative of function f at $x = 3$

$f''(x)$ = derivative of $f'(x)$

    or the 2nd derivative of $f(x)$

$f'''(x)$ = derivative of $f''(x)$

    or the 3rd derivative of $f(x)$

$f^n(x)$ = the *nth* derivative of f.

Given equation y

y' = derivative of equation y

$y'(3)$ = derivative of y at $x = 3$

y'' = derivative of y'

    or the 2nd derivative of y

y''' = derivative of y''

    or the 3rd derivative of y

y^n = *nth* derivative of y

Use of the prime marks is also known as Lagrange notation.

Leibniz Notation

$$\frac{dy}{dx} = \text{derivative of } y \text{ with respect to } x$$

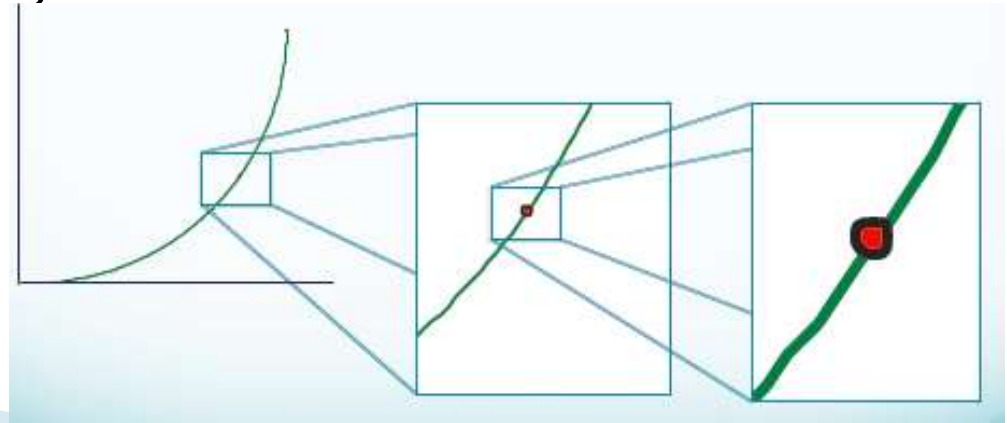$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = \text{2nd derivative of } y$$

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3} = \text{3rd derivative of } y$$

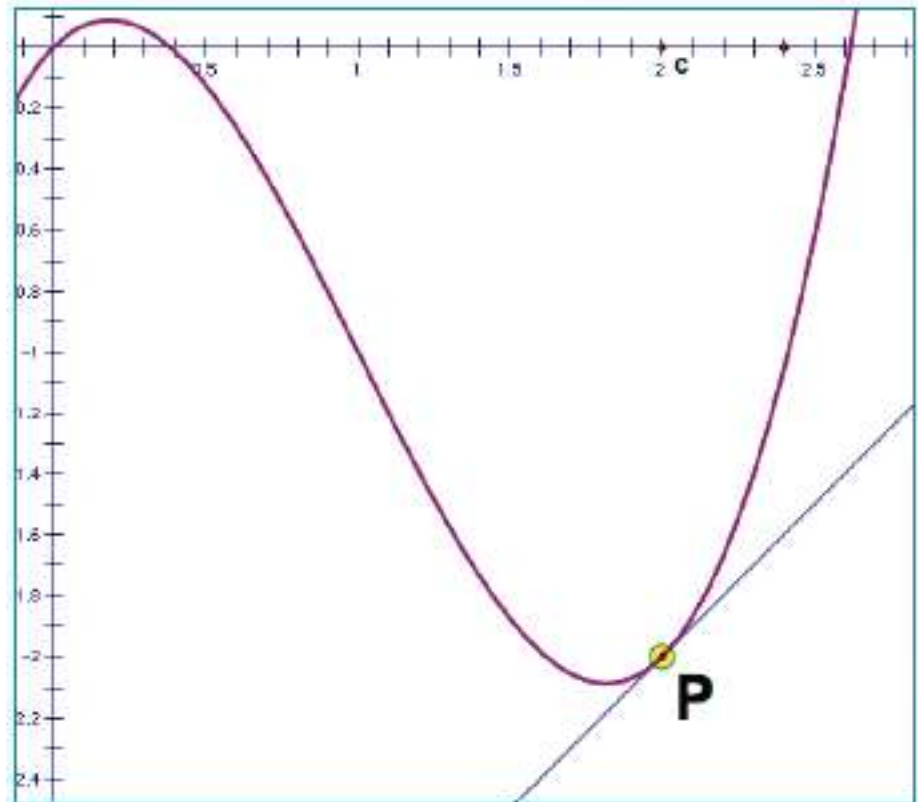$$f'(x) \approx y' \approx \frac{dy}{dx} = \text{1st derivative}$$

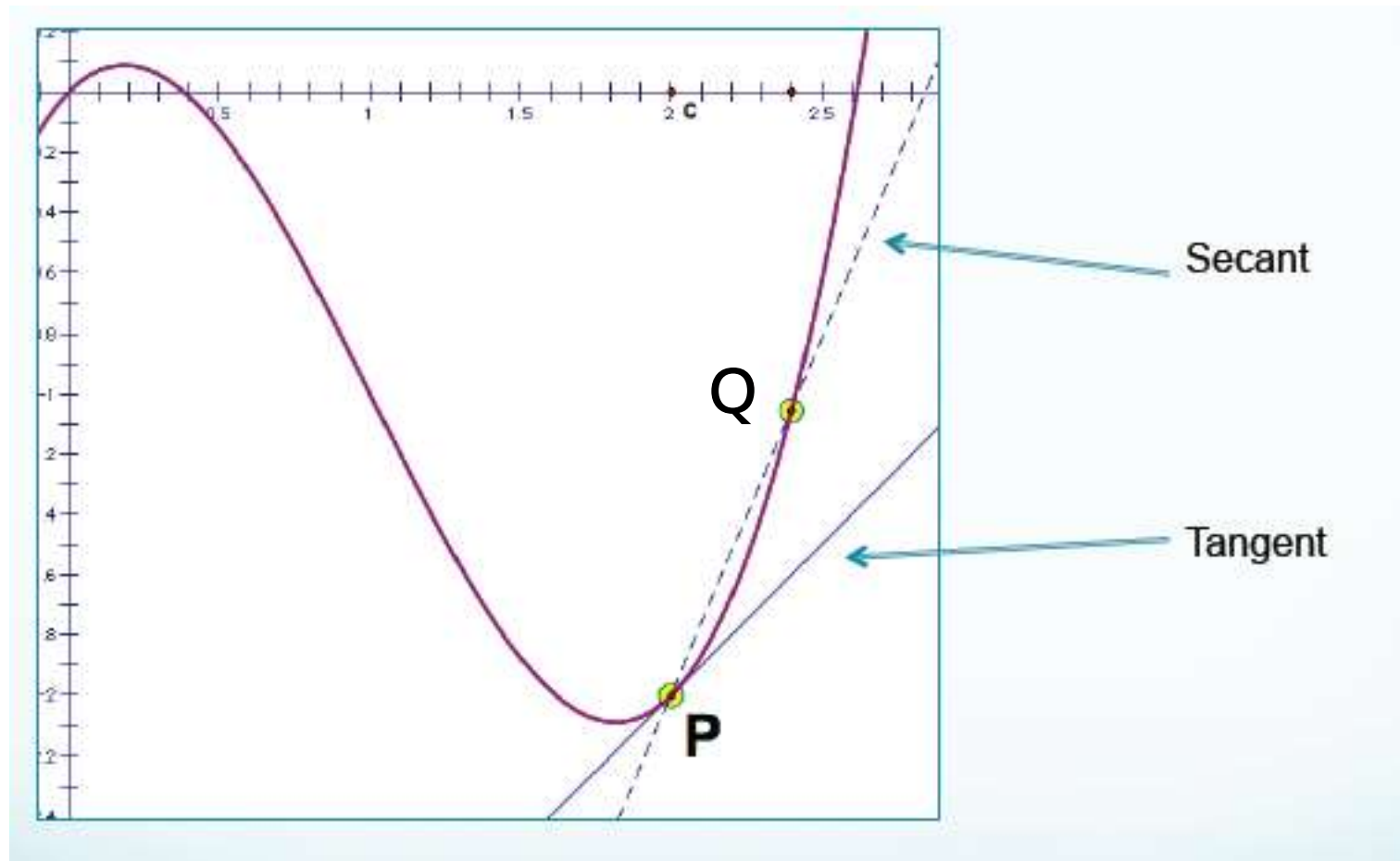$$f''(x) \approx y'' \approx \frac{d^2y}{dx^2} = \text{2nd derivative}$$

How to find the derivative

▸ If graph is linear, use the slope formula

▸ If graph is curved, local linearization could be a way to approximate

▸ **Local Linearity**: at a point, a curve behaves indistinguishably from a straight line; when you zoom in a curve, it looks like a line
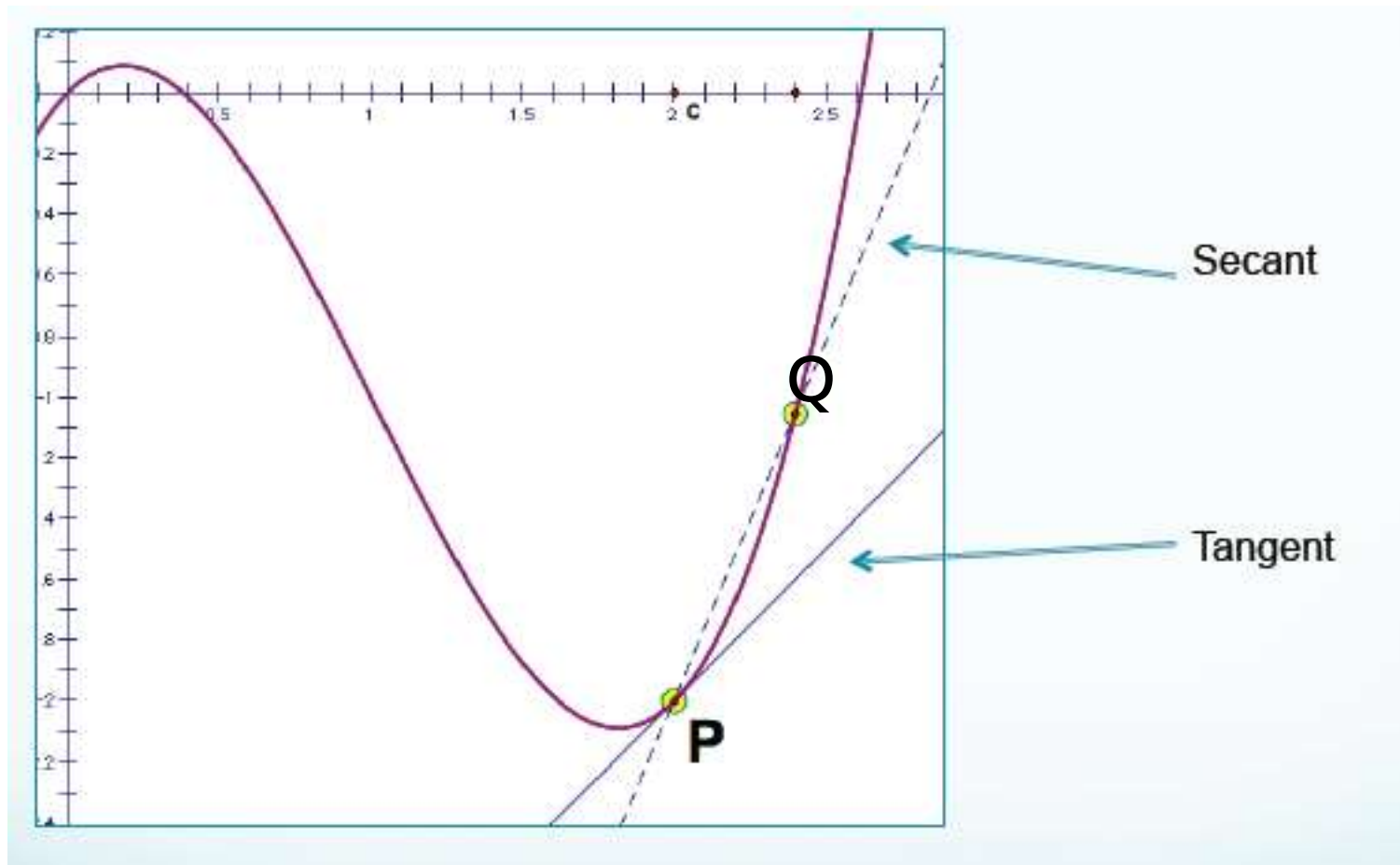
The derivative of a function at a point is the slope of the tangent line at that point.
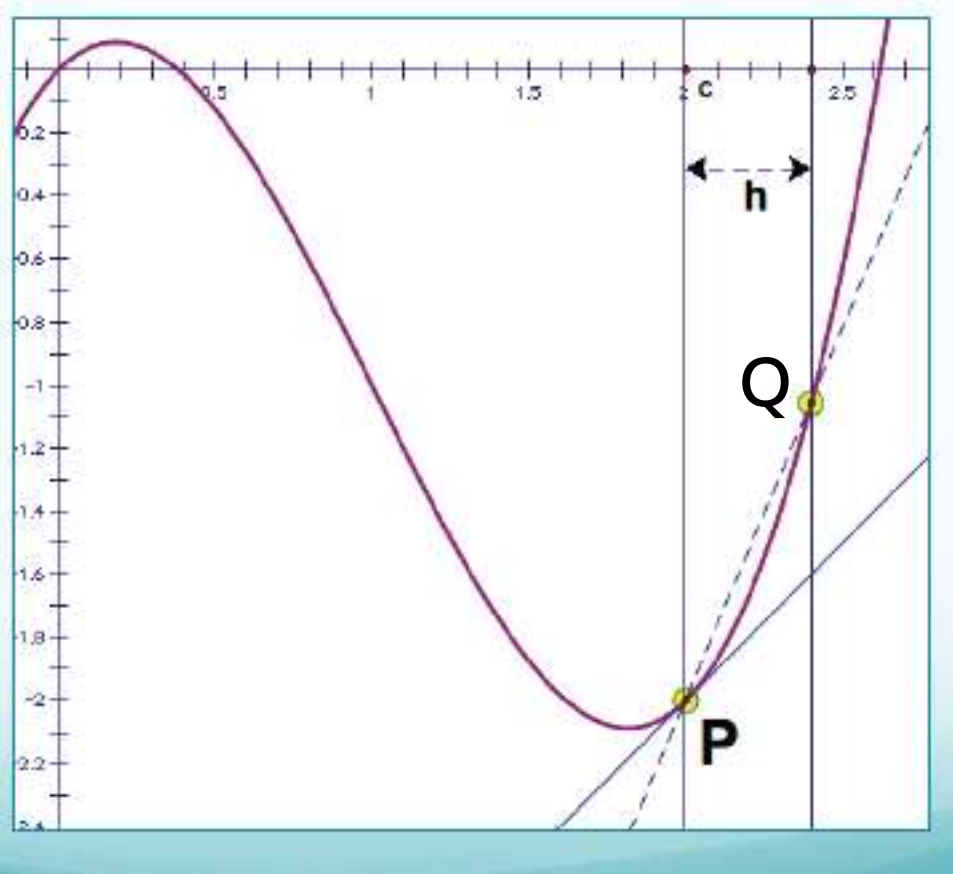
▸ Let's pick a second point and draw a secant line and use its slope to estimate the tangent's slope.

As Q gets closer to P, the slope of the secant will approach the slope of the tangent line. (As seen on sketchpad demo)

P(x, f(x))   Q(x + h, f(x + h))

Slope of secant $= \dfrac{f(x+h)-f(x)}{x+h-x}$

Slope of tangent $=$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{x+h-x} =$$

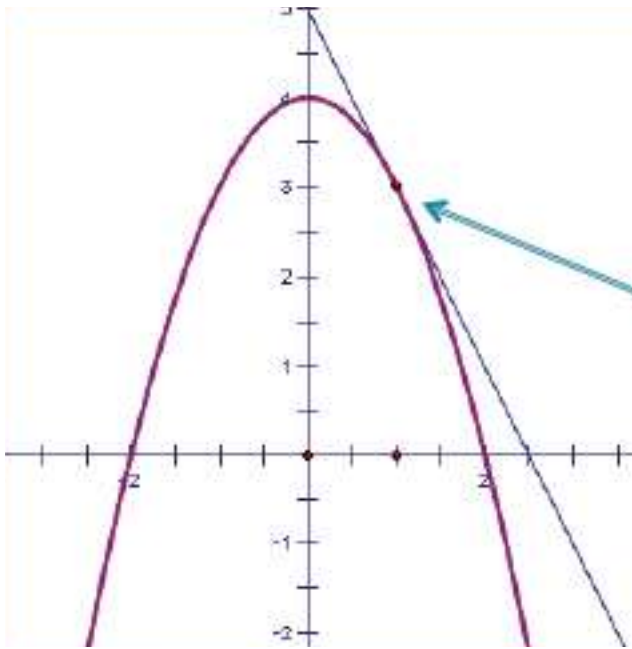$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

$$f(x) = 4 - x^2 \quad \text{Find f'(1)}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

What does f'(1) = −2 mean?

The slope of the tangent line at x = 1 is −2.

If you have a point and the slope, you can write the equation for the tangent line.

Or the equation of the line normal to the tangent line at x = 1.

You try: $y = \dfrac{3}{x}$

- Find a) y'(3)
- b) write the equation of the tangent line and normal line at x = 3