Derivatives and Integration of Series

Section 9.9 continued

Derivative of a Series

$$\frac{d}{dx}\left(3+3x+3x^2+3x^3\right)$$

Notice our derivative has one less term than the original series.

 $\frac{d}{dx}(3+3x+3x^2+3x^3+\cdots 3x^n+\cdots)$

$$\frac{d}{dx}\sum_{n=0}^{\infty}3x^n = \sum_{n=1}^{\infty}3nx^{n-1}$$

Increased for lost of 1st term whose derivative is zero Whatever is valid for a polynomial is usually good for a series.

Use Power and Chain rules.

n is a constant, x is a variable.

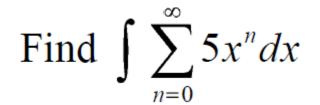
Interval and radius of convergence are the same.

$$\frac{d}{dx}\sum_{n=0}^{\infty} 4(2x)^n$$

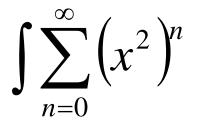
The endpoints may or may not be included.

Integrals of Series $\int 3+3x+3x^2+3x^3 dx$

 $\int 3 + 3x + 3x^2 + 3x^3 + \dots + 3x^n + \dots dx$



 $\int \sum_{n=1}^{\infty} (-1)^n x^{2n}$ n=0



Now let's use derivatives and integrals to find series for functions not easily written as $\frac{a}{1-r}$.

$$f(x) = \frac{1}{\left(1 - x\right)^2}$$

Express the function as a series $f(x) = \ln(1-x)$

Express the function as a series $f(x) = \tan^{-1} x$

Express the function as a series

$$f(x) = \frac{x^2}{\left(1+x\right)^2}$$

Series Manipulation Techniques

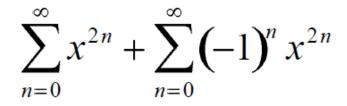
Substitute into a known Series

Yesterday we proved that $\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

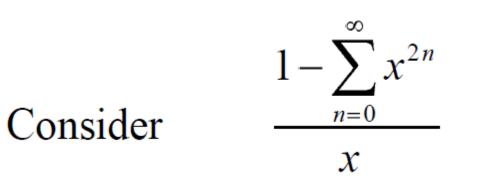
Find a series for
$$g(x) = \tan^{-1}(x^4)$$

Expand and Cancel

Consider



Same idea as the Telescoping Test



What you cannot do

> 2

$$\left(\sum_{n=0}^{\infty} c_n x^n\right)^2 = (a_1 + a_2 + a_3 + \cdots) (a_1 + a_2 + a_3 + \cdots)$$

We cannot square a series.

Doing so would require infinite foiling or methods beyond the scope of this class.