Differential Equations

AP EXAM REVIEW

Compare and Contrast

Solve $\int 3x^2 \ dx$ with the initial condition F(1) = 4

Solve $\dfrac{dy}{dx} = 3x^2 \qquad y(1) = 4$

This is called a differential equation. You solve it the same way. Your solution is $y =$ equation.

Differential equations can be used to solve problems when you know something about the rate of change. For example, you can predict the future position of planets based on their velocities or the number of bacteria in the future based on their rate of growth.

Euler's Method

Begin with starting point (x_o, y_0).

Determine the step size Δx.

Interate:

Original x　　　　Original y

Next x　　　　Next y = original y + $\frac{dy}{dx} \cdot \Delta x$

Example: y'=x+2, initial pt (0, 0), Δx = 0.5, find y(3).

Slope Fields Skills

Match differential equation to slope field.

Match solution to differential equation to slope field.

Manually construct a portion of a slope field.

Pay attention to where the slope is zero.

Pay attention to where the slope is positive or negative.

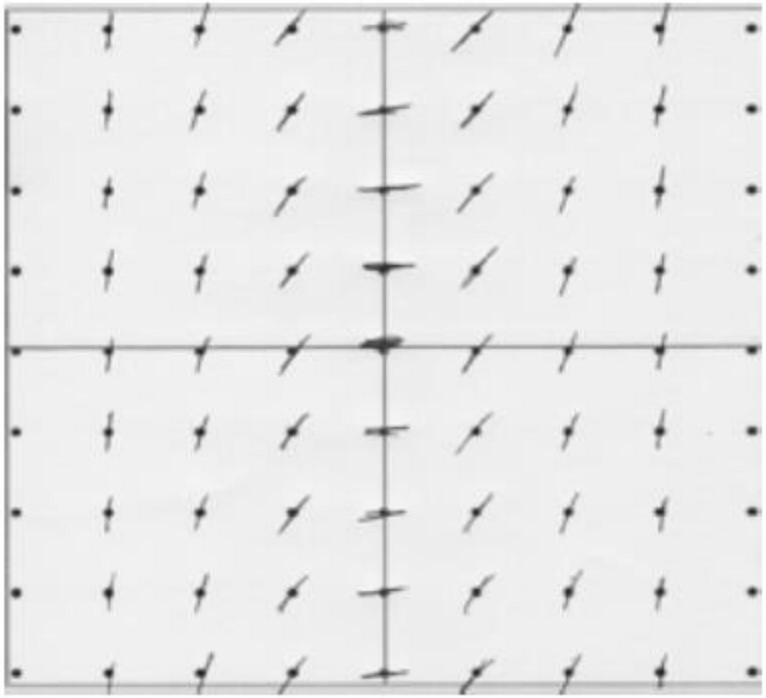
Look at the slope lines on the x-axis (where the y-value is 0).

Look at the slope lines on the y-axis (where the x-value is 0).

A <u>slopefield</u> is a tool to learn the characteristics of the solution to an indefinite integral without actually knowing the antiderivative.

You must be able to
1. Draw a slopefield
2. Indicate a particular solution on a slopefield
3. Answer questions about a slopefield

Solve $\dfrac{dy}{dx} = x^2$



Graphed solutions:
- Include the given point
- Do not cross discontinuities
- Do not cross slope lines

Slopefields should clearly show

Positive vs. negative slopes
Slopes of 1 and –1
Whether slopes are increasing or decreasing
Zero and undefined slopes
Do NOT plot at points where dy/dx is indeterminate (0/0)
Symmetry

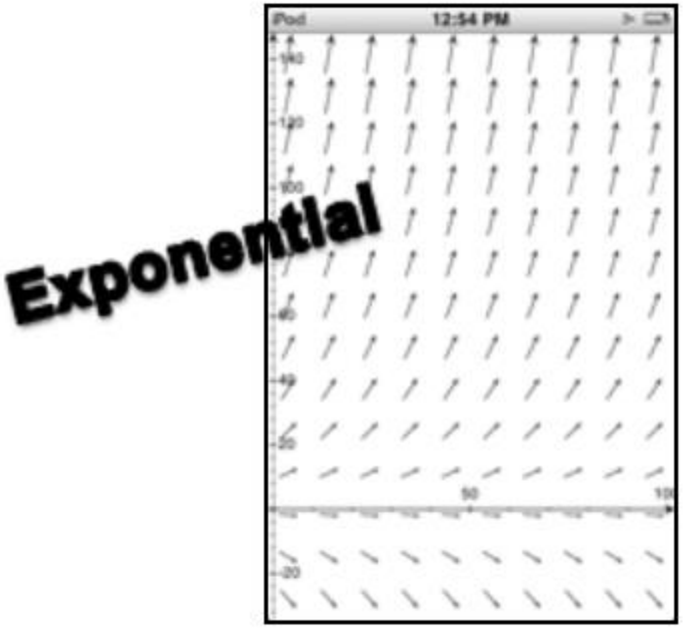
Specific solutions should…

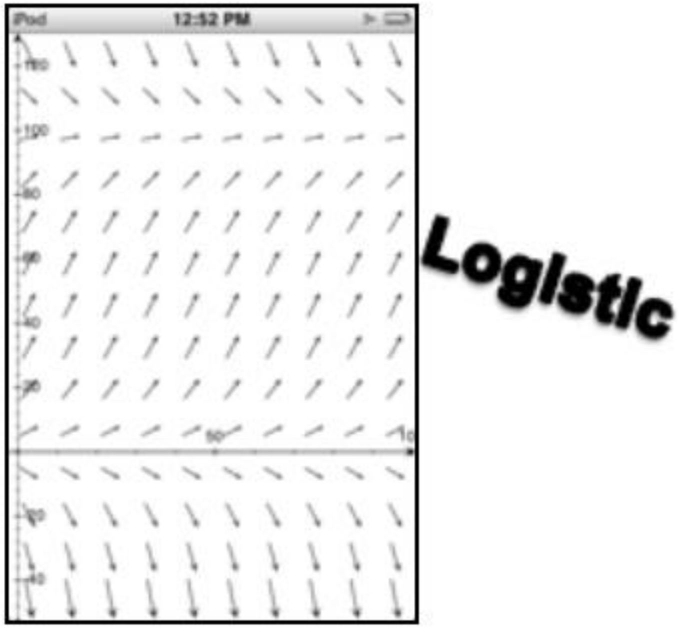
Show point plotted at the given initial condition
Flow through the slopefield field
Never, never, never cross a slope line!
Never cross a gap, DNE, or vertical tangent.

Specific solutions are to be the largest, *continuous* solution that goes through the slopefield and the given initial condition.

Exponential and Logistic Functions

M is carrying capacity.
Rate is greatest at ½ M.





$$\frac{dP}{dt} = kP$$

$$P = P_0 e^{kt}$$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

$$P = \frac{M}{1 + Ae^{-kt}}$$

Newton's Law of Cooling

The rate of change of y is proportional to the difference between y and the ambient temperature.

If the room temperature is 60∘,

$\frac{dy}{dt} = k(y - 60)$ or $y - 60 = (y_0 - 60)e^{kt}$

Separable Differential Equations

1. Solve $\dfrac{dy}{dx} = \dfrac{x}{y}$ $\qquad f(2) = 4$

Separate the x's and y's

Integrate each side

Solve for *c* if you can

Solve for *y*

Solve $\dfrac{dy}{dx} = x^2 y \qquad y(0) = 2$

Separate

Integrate

Solve for y

Get rid of | | by using $\pm$

Consolidate constants

Now, solve for *A*

$$y = Ae^{\frac{x^3}{3}}$$

Use given initial condition

Final answer