AP Calculus BC Review 1 Name: _____

Non Calculator (Calculator permitted on Free Response)

C 1. Evaluate $\lim_{h\to\frac{1}{2}} \frac{e^h - \sqrt{e}}{h - \frac{1}{2}}$ $e^{\frac{1}{2}}$ L'Hopital

a. e^2 b. $e^2 - 1$ c. $\sqrt{e}$ d. e e. The limit does not exist.

1 2. What is the coefficient of x^3 in the Taylor series for $x\sin x^2$ centered at x = 0?

$\sin x = x - x^3/3!$

$\sin x^2 = x^2 - x^6/3!$

$x \sin x^2 = x^3 - \frac{x^7}{3!}$

a. 1 b. $\frac{2}{3}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$ e. $\frac{1}{6}$

A 3. The graph of $y = x^3 + 21x^2 - x + 1$ is concave down for

$y' = 3x^2 + 42x - 1$

$y'' = 6x + 42 = 0$

$x + 7 = 0$

a. x < -7 b. −7 < x < 7 c. all x d. x > 7 e. x < -7 and x > 7

D 4. The slope of the tangent to the curve $2y^2x^3 - 5x^2y = 18$ at the point $(1, 2)$ is

$4y\frac{dy}{dx}x^3 + 6y^2x^2$

$-10xy - 5x^2\frac{dy}{dx} = 0$

$8\frac{dy}{dx} + 24 - 20 - 5\frac{dy}{dx}$

$3\frac{dy}{dx} = -4$

a. $-\frac{4}{5}$ b. $-\frac{3}{2}$ c. 0 d. $-\frac{4}{3}$ e. −1

e 5. $\frac{d}{dx}\int_0^{x^2} \cos(t^2)dt$ $f(x^2) - f(0)$

$2x f'(x^2$

a. $\cos(x^4)$ b. $\cos(x^2)$ c. $2x\cos(x^2)$ d. $x\cos(x^4)$ e. $2x\cos(x^4)$

Free Response: A particle moves along the x-axis so that its velocity at time t for $0 \le t \le 6$, is given by $v(t) = 4\sin\left(\frac{t^2}{2} - 2t + 2\right)$. At time t = 0, the particle is at x = 1.

a. Find the velocity and acceleration of the particle at t = 4. $v(4) = 3.637$ $a(4) = -3.32$

b. Is the speed of the particle increasing or decreasing at time t = 4. decreasing

c. Find the time when the particle first changes direction. $t = 4.507$

d. Describe the movement of the particle near time t = 2. stopped at $t = 2$

AP Calculus BC Review 2 Name: Key
Calculator

1. Find the values of x for which the function $y = x^6 + 3x^5 - \frac{15}{2}x^4 - 40x^3 - 60x^2 + 8x + 5$ has inflection points.

$y' \; 6x^5 + 15x^4 - 30x^3 - 120x^2 - 120x + 8$
$y'' = 30x^4 + 60x^3 - 90x^2 - 240x - 120 = 0$

a. There are none. b. x = -2, 2 c. x = -1, 0, 1 d. x = -2, 0, 2 e. x = 0

$x^4 + 2x^3 - 3x^2 - 8x - 4 = 0$

2. A curve is given parametrically by the equations $x = 3t - t^3$ and $y = 3t^2$. The length of the arc from t = 0 to t = 2 is

$\int_0^2 \sqrt{(3-3t^2)^2 + (6t)^2}\, dt$

a. 6 b. 8 c. 10 d. 12 e. 14

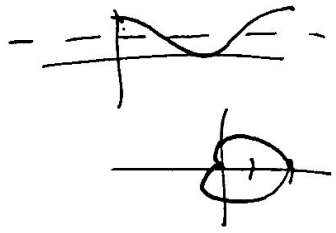
3. Using the trapezoid rule, approximate the area under the curve $f(x) = x^2 + 4$ between x = 0 and x = 3 using 6 subintervals.

Used Table

a. 18.875 b. 20.938 c. 21 d. 21.125 e. 23.375

$A = \frac{1}{2}\left(\frac{1}{2}\right)\left[f(0) + 2(f(.5)) + 2f(1) + 2f(1.5) + 2f(2) + 2f(2.5) + f(3)\right]$
$\frac{1}{4}\left[4 + 2(4.25) + 2(5) + 2(6.25) + 2(8) + 2(10.25) + 13\right]$

4. Which of the following gives the area of the region enclosed by the graph of the polar curve $r = 1 + \cos\theta$?



a. $\int_0^{\pi} 1 + \cos^2\theta\, d\theta$ b. $2\int_0^{\pi} (1+\cos\theta)^2\, d\theta$ c. $\int_0^{2\pi} 1 + \cos\theta\, d\theta$

d. $\int_0^{2\pi} (1+\cos\theta)^2\, d\theta$ e. $\frac{1}{2}\int_0^{2\pi} 1 + \cos^2\theta\, d\theta$

5. The average value of the function $f(x) = \frac{2x}{x^2 - 4}$ on the interval [5, 8] is

[a, b]

a. 0.350 b. 1.050 c. 0.743 d. 0.248 e. 0.201

$\frac{\int_5^8 \frac{2x}{x^2-4}}{3}$ $\frac{1}{b-a}\int_a^b f(x)\, dx$

AP Calculus BC Review 3
Non-calculator

Name: Key

A 1. $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{2n} =$ $\frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}$

a. $\frac{1}{3}$ b. $\frac{1}{2}$ c. 1 d. 2 e. ∞

E 2. The velocity of a runner in feet per second as a function of time is given below.

t	0	1	2	3	4
v(t)	5	10	12	11	9

The approximate acceleration of the runner at time t=2 is

a. $3\frac{ft}{sec^2}$ b. $-3\frac{ft}{sec^2}$ c. $-2\frac{ft}{sec^2}$ d. $-\frac{1}{2}\frac{ft}{sec^2}$ e. $\frac{1}{2}\frac{ft}{sec^2}$

$\frac{11-12}{3-2} = -1$

$\frac{12-10}{2-1} = 2$ Not the

C 3. $\lim_{x\to\infty}\frac{x^2-3x+7}{\sqrt{4x^4-3x^3+2x^2}}$

a. The limit does not exist. b. 1 c. ½ d. ¼ e. 0

$\frac{11-10}{3-1} = \frac{1}{2}$

C 4. $\int_1^{\infty} x^{-\frac{5}{4}}dx$ $-4x^{-\frac{1}{4}} = -4\left(\frac{1}{\sqrt[4]{x}}\right]_1^{\infty} = 4$

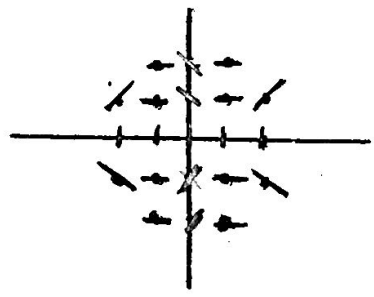
a. $\frac{5}{4}$ b. $\frac{1}{4}$ c. 4 d. -4 e. nonexistent

A 5. Find the slope of the curve $x^2-4xy-y^2=-4$ at the point (3, 1).

a. 1/7 b. 7/2 c. 3/2 d. 2/3 e. 2/7

Free Response: Consider the equation $\frac{dy}{dx}=\frac{x^2-1}{y}$. $\frac{3}{2}$

a. On the axes provided, sketch a slope field for the given differential equation at the 12 points indicated.



b. Given that f(3) = 4, find the equation of the line tangent to the graph at x = 3.

c. Find the particular solution to the differential equation when f(3) = 4.

d. Use Euler's Method to approximate f(5) with $\Delta x = 1$.

$2x-4y-4x\,dy-2y\,dy=0$

$2-4-12\,dy-2\,dy=0$

$-14\,dy=2$

$\frac{dy}{dt} = \frac{1}{7}$

(b) $y=4+\frac{8}{4}(x-3) = 4+2(x-3)$

(c) $\int y\,dy = \int(x^2-1)dx$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 - x + C$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 - x + 2$

$y = \sqrt{\frac{2}{3}x^3 - 2x + 4}$

(d) $y = 4+2(1) = 6$ (4,6)

$y = 6+\frac{15}{16}(1) = 8.5$

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AP Calculus BC Review 4 Name: _____

Non Calculator

C 1. Determine the number of points where the tangent lines to $x^2+4y^2=16$ are horizontal.

a. 0 b. 1 c. 2 d. 3 e. 4

E 2. $\int_0^4 |2x-6|\,dx$ not D $\int_0^4 (2x-6)\,dx$

a. 0 b. 2.5 c. 5 d. 8 e. 10

D 3. Determine the slope of the line normal to the curve $x^3+xy^2=10y$ at $(2, 1)$

a. 0 b. 2 c. $-\frac{7}{3}$ d. $-\frac{6}{13}$ e. $\frac{1}{2}$

$3x^2+y^2+2xy\,dy/dx = 10\,dy/dx$

$12+1+4\,dy/dx = 10\,dy/dx$

$13 = 6\,dy/dx$

$\frac{13}{6}$

D 4. What are the first four nonzero terms in the power series expansion of e^{-4x} about $x=0$?

a. $1+x+\frac{x^2}{2}+\frac{x^3}{3}$ b. $1-4x+8x^2-32x^3$ c. $1-4x-2x^2-\frac{2}{3}x^3$

d. $1-4x+8x^2-\frac{32}{3}x^3$ e. $1-4x+8x^2-\frac{64}{3}x^3$

$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$

$1-4x+\frac{16x^2}{2}-\frac{64x^3}{6}$

D 5. An object has velocity given by $v(t)=4t+e^{t-2}$. At $t=2$, the object's position is $y(2)=3$. The function $y(t)$ describing the object's position for any time $t>0$ is

a. $y(t)=9t-15$ b. $y(t)=2t^2+e^{t-2}+9$ c. $y(t)=9t-9$

d. $y(t)=2t^2+e^{t-2}-6$ e. $y(t)=9t-6$

$y = 2t^2+e^{t-2}+C$

$3 = 8+1+C$

$-6 = C$

Free Response: The derivatives of f(x) at x = 0 and given by $f^{(n)}(0)=\frac{(n+1)!}{3^n}$.

a. Show that the Taylor series for f(x) at x = 0 is $\sum_{n=0}^{\infty}\frac{(n+1)x^n}{3^n}$.

$\frac{(n+1)!}{3^n}\,\frac{(x^n)}{n!}$

b. What is the radius of convergence of the series?

$\lim_{n\to\infty}\left|\frac{\frac{(n+2)}{3^{n+1}}x^{n+1}}{\frac{n+1}{3^n}x^n}\right| = \lim_{n\to\infty}\left|\frac{(n+2)x}{(n+1)3}\right| = \left|\frac{1}{3}x\right| < 1$

$|x| < 3$ radius = 3

AP Calculus BC Review 5 Name: _____
Calculator

D 1. The expression $\frac{1}{10}\left[\left(\frac{1}{10}\right)^2+\left(\frac{2}{10}\right)^2+\left(\frac{3}{10}\right)^2+\ldots+\left(\frac{10}{10}\right)^2\right]$ is a Riemann sum approximation of which integral?

a. $\int_1^{10} x^2dx$ b. $\int_1^{10}\left(\frac{1}{x}\right)^2 dx$ c. $\int_0^{1}\left(\frac{1}{x}\right)^2 dx$ d. $\int_0^{1} x^2dx$ e. $\frac{1}{10}\int_0^{1} x^2dx$

D 2. The velocity vector of a particle moving in the xy-plane is given by $\langle 2\sin t, 3\cos t\rangle$ for $t \geq 0$. At t = 0, the particle is at the point (1, 1). What is the position vector at t = 2?

a. $\langle 0,3\rangle$ b. $\langle 1.819, -1.248\rangle$ c. $\langle 1.735, -0.532\rangle$ d. $\langle 3.832, 3.728\rangle$ e. $\langle 1.832, -1.728\rangle$

C 3. Water flows into a pond at a rate of $300\sqrt{t}$ gallons/hour and flows out at a rate of 400 gallons/hour. After 1 hour there is 10,000 gallons of water in the pond. How much water is in the pond after 9 hours?

$10{,}000 + \int_1^9 (300\sqrt{t} - 400)$

a. 10,000 b. 11,000 c. 12,000 d. 14,000 e. 16,000

D 4. Let $f(x) = \int_{\frac{2\pi}{3}}^{x^2} \cos t\,dt$. Which of the following is a critical point of f?

$f'(x)$ $\cos x^2 = 0$

a. π b. $\frac{\pi}{2}$ c. $\sqrt{\pi}$ d. $\sqrt{\frac{\pi}{2}}$ e. No critical point exists.

C 5. If $\int_a^b f(x)dx = 2a - 3b$, then $\int_a^b f(x)+3dx =$

a. 2a – 3b + 3 b. 3b – 3a c. –a d. 5a – 6b e. a – 6b

Free Response: The region R is enclosed by the graphs of $f(x) = \cos(x-0.4)$ and $g(x) = x^2$.

a. Find the area of the region R. $A = \int_{-.684}^{.9291} (\cos(x-.4) - x^2)dx = 1.015$

b. Find the volume generated by revolving the region R about the line y = 3. $\pi\int_{-.684}^{.9291} [(3-x^2)^2 - [3-\cos(x-.4)]^2 dx = 5.025\pi$

c. The region R is the base of a solid. Each cross section in the plane perpendicular to the x-axis is a rectangle. The width is three times longer than the base. Find the volume of the solid.

$2.302 = \int_{-.684}^{.9291} [\cos(x-.4) - x^2] \cdot 3[\cos(x-.4) - x^2]$ 15.786

base $l = 3$base

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