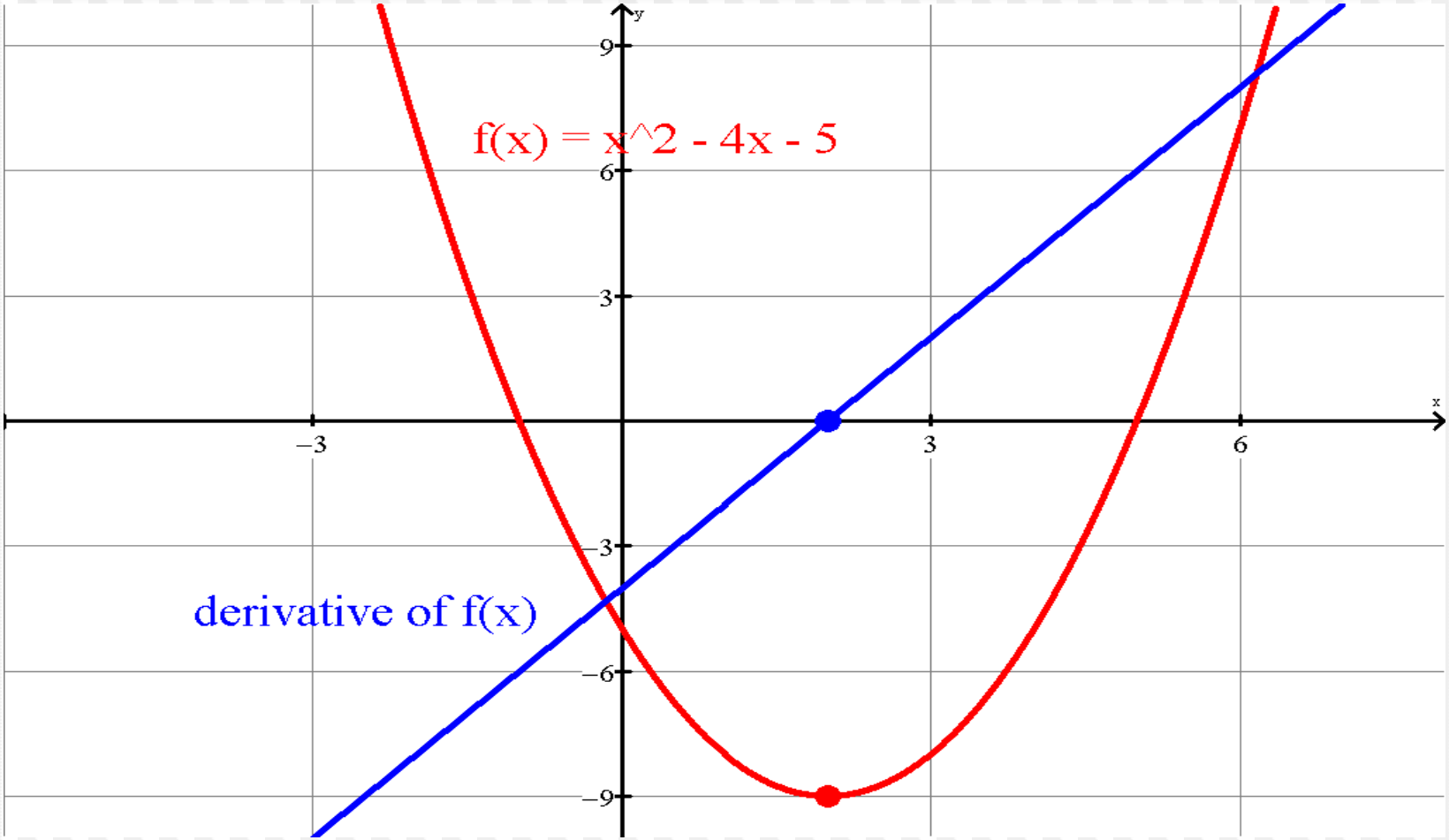
Connecting the graphs of a function and its derivative

Lesson Objectives

1. From a graph of a function, sketch its derivative

2. From a graph of a derivative, graph an original function

Example #1



Example #1

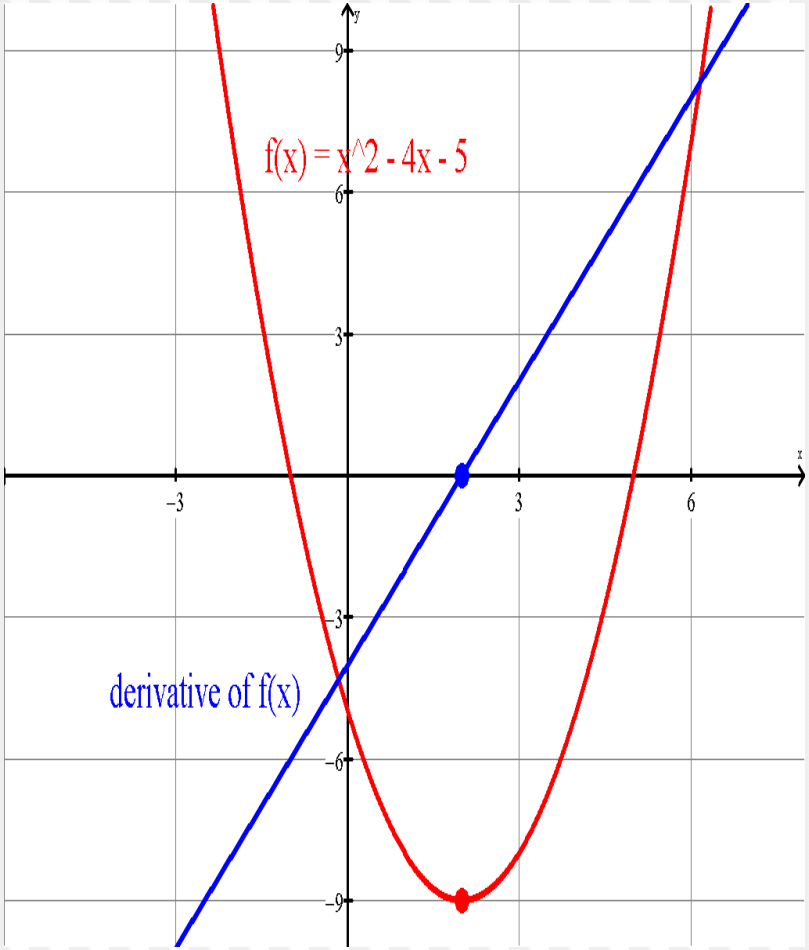
Points to note:

(1) the f(x) has a minimum at x=2 and the derivative has an x-intercept at x=2

(2) the f(x) decreases on (-∞,2) and the derivative has negative values on (-∞,2)

(3) the f(x) increases on (2,+∞) and the derivative has positive values on (2,+∞)

(4) the f(x) changes from decrease to increase at the min while the derivative values change from negative to positive

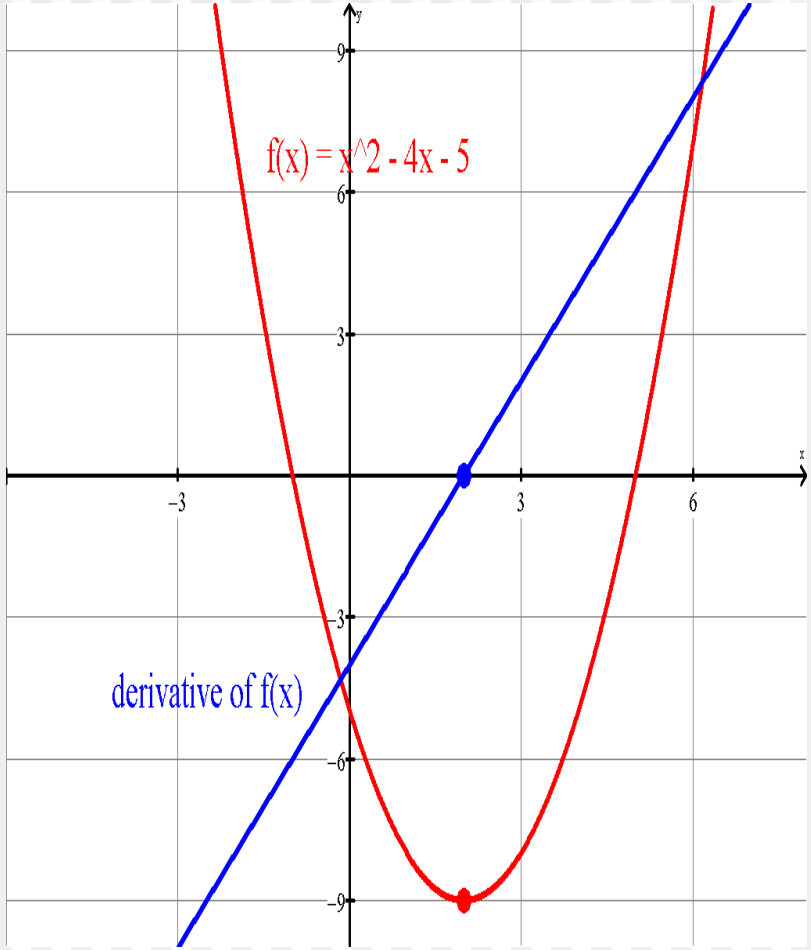


Example #1

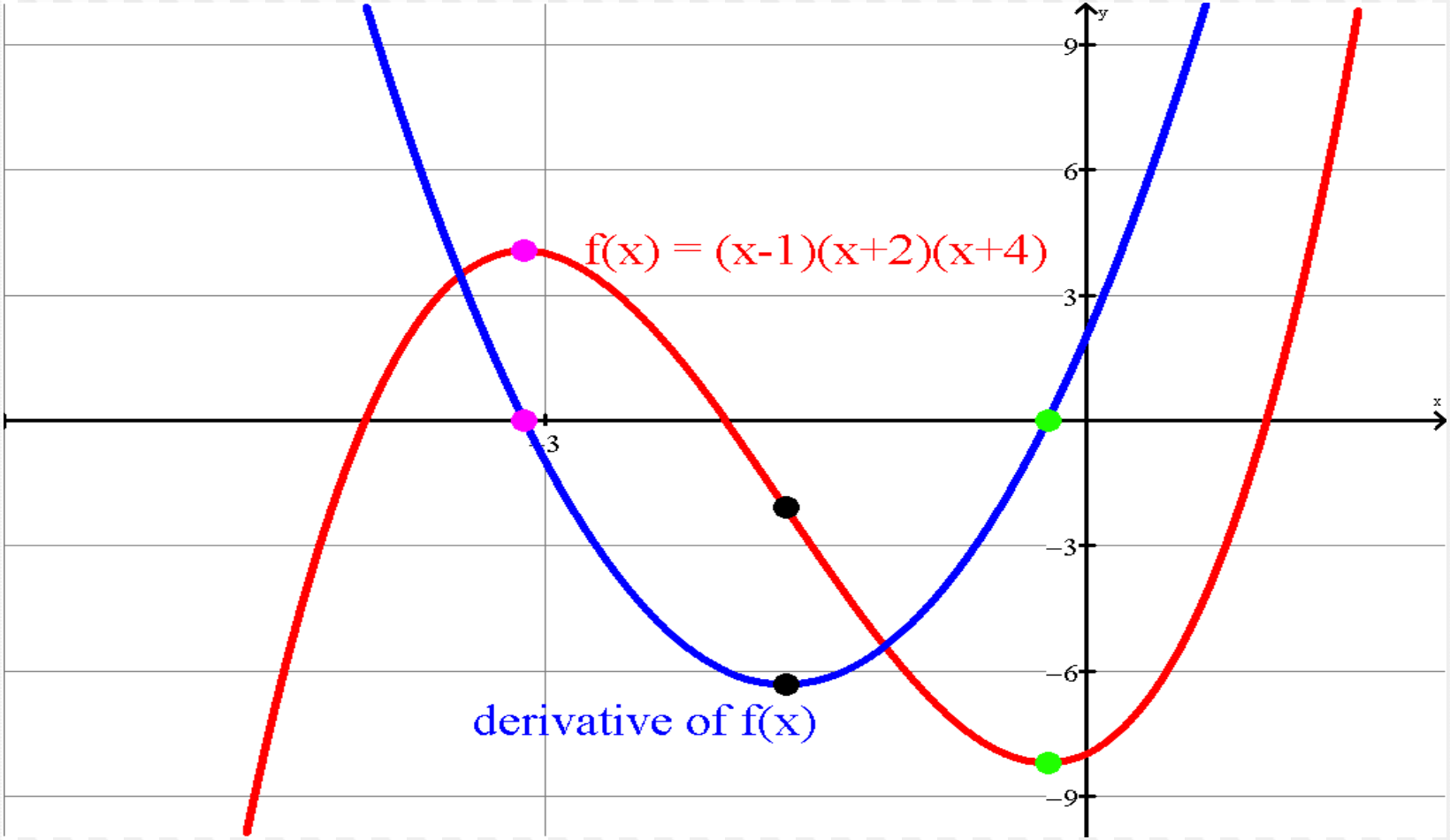
Points to note:

(5) the function is concave up and the derivative is increasing

(6) The second derivative of f(x) is positive everywhere

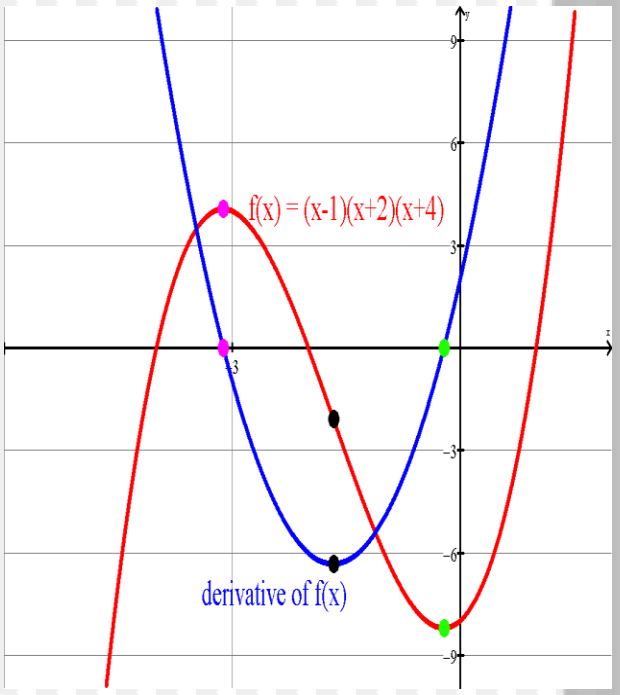


Example #2



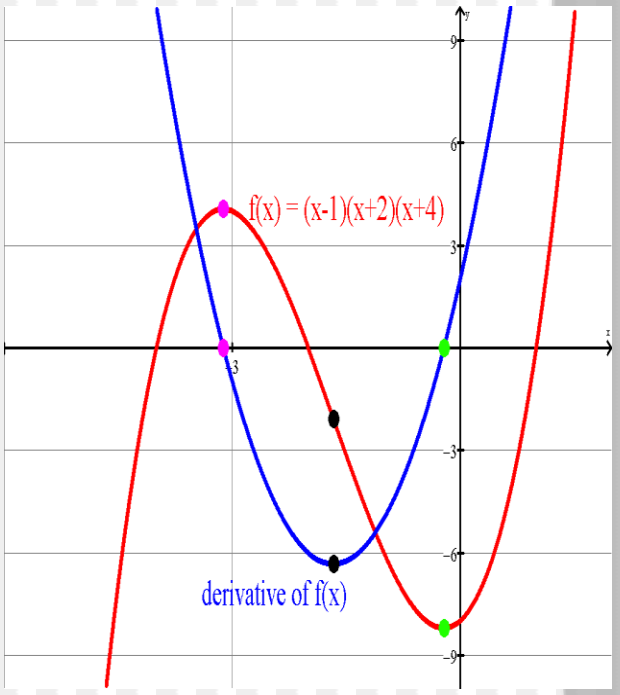
Example #2

- f(x) has a max. at x = -3.1 and f `(x) has an x-intercept at x = -3.1
- f(x) has a min. at x = -0.2 and f `(x) has a root at –0.2
- f(x) increases on (-∞, -3.1) & (-0.2, ∞) and on the same intervals, f `(x) has positive values
- f(x) decreases on (-3.1, -0.2) and on the same interval, f `(x) has negative values
- At the max (x = -3.1), f(x) changes from increasing to decreasing ➔ the derivative changes from positive values to negative values
- At a the min (x = -0.2), f(x) changes from decreasing to increasing ➔ the derivative changes from negative to positive



Example #2

- At the max (x = -3.1), f(x) changes from increasing to decreasing ➔ the derivative changes from positive values to negative values
- At a the min (x = -0.2), f(x) changes from decreasing to increasing ➔ the derivative changes from negative to positive
- f(x) is concave down on (-∞, -1.67) while f `(x) decreases on (-∞, -1.67)
- f(x) is concave up on (-1.67, ∞) while f `(x) increases on (-1.67, ∞)
- The concavity of f(x) changes from CD to CU at x = -1.67, so f(x) has an inflection point at x = -1.67, while the derivative has a min. at x = -1.67



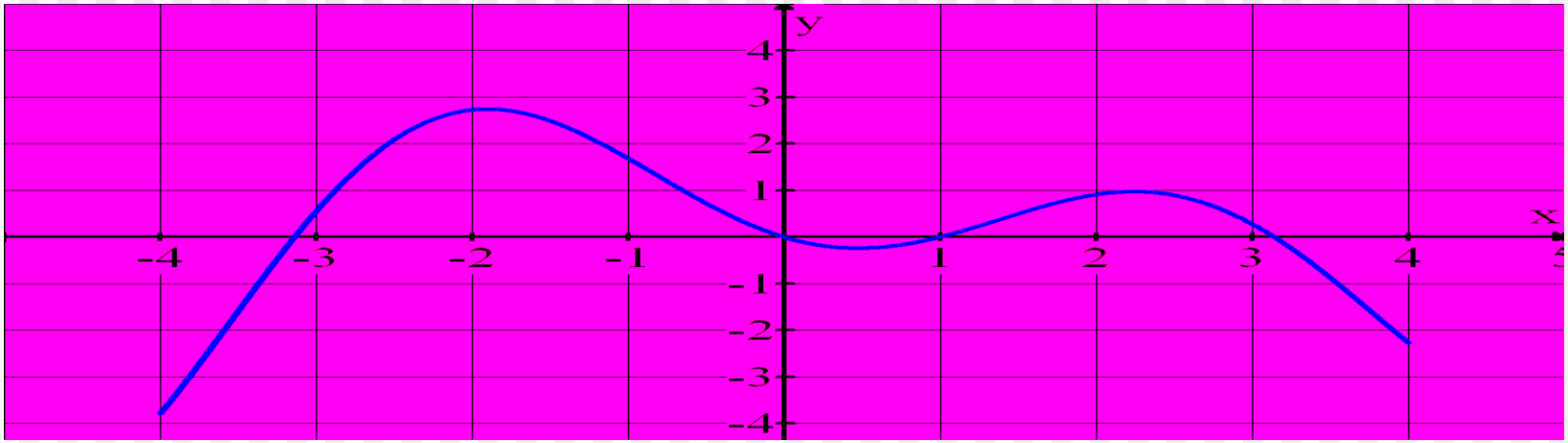
The big derivative puzzle

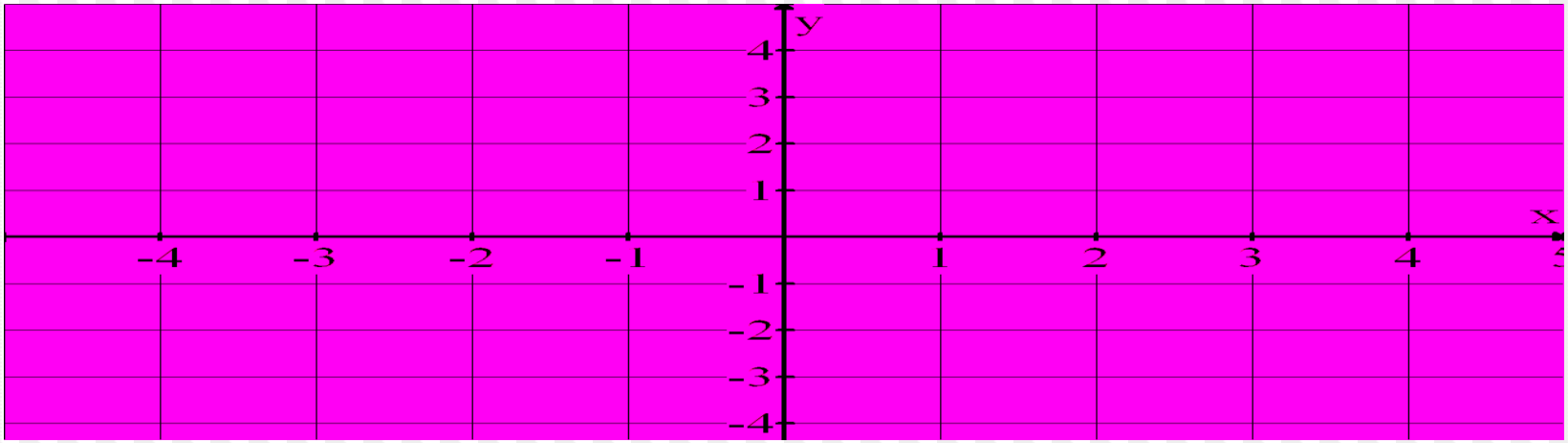
Let's try to match a function with its derivative.

Matching Function Graphs and Their Derivative Graphs

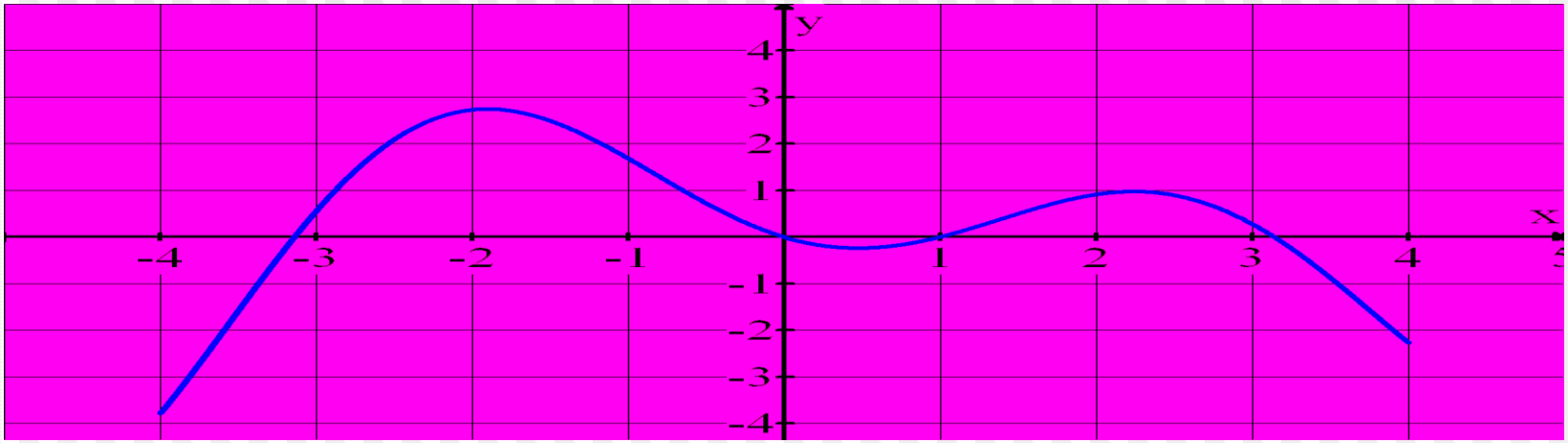
- To further visualize the relationship between the graph of a function and the graph of its derivative function, we can run through some exercises wherein we are given the graph of a function ➔ can we draw a graph of the derivative and vice versa

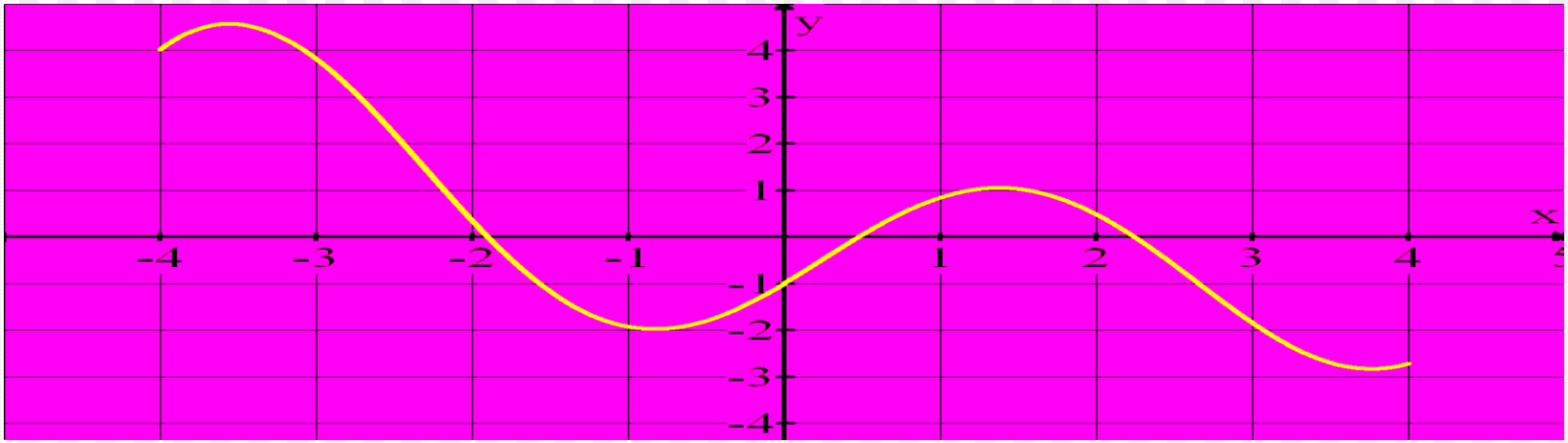
Matching Function Graphs and Their Derivative Graphs



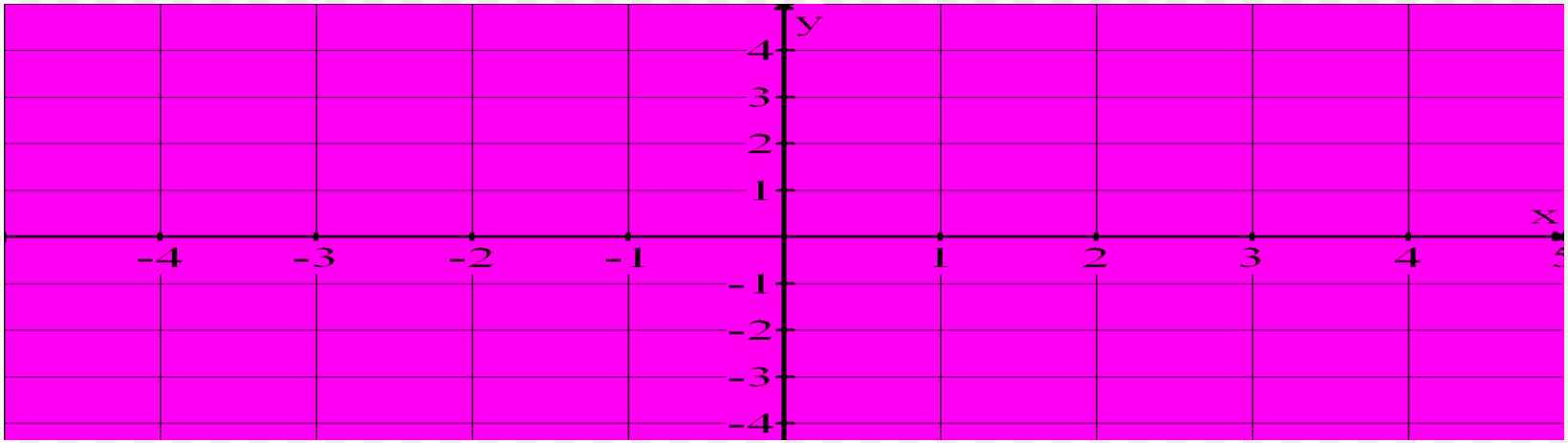


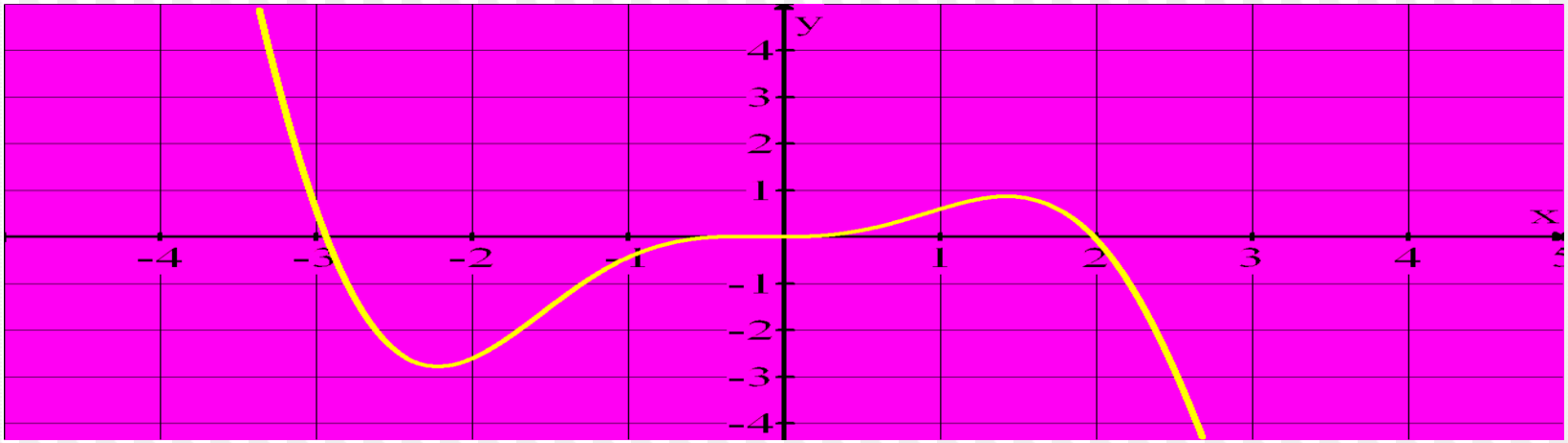
Matching Function Graphs and Their Derivative Graphs - Answer





Matching Function Graphs and Their Derivative Graphs – Working Backwards





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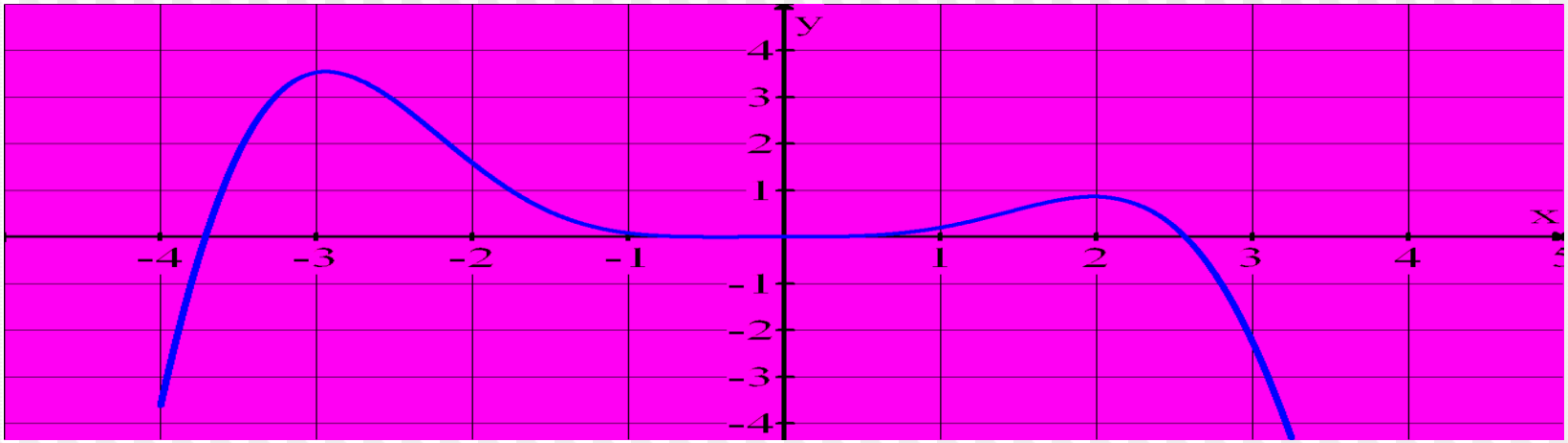
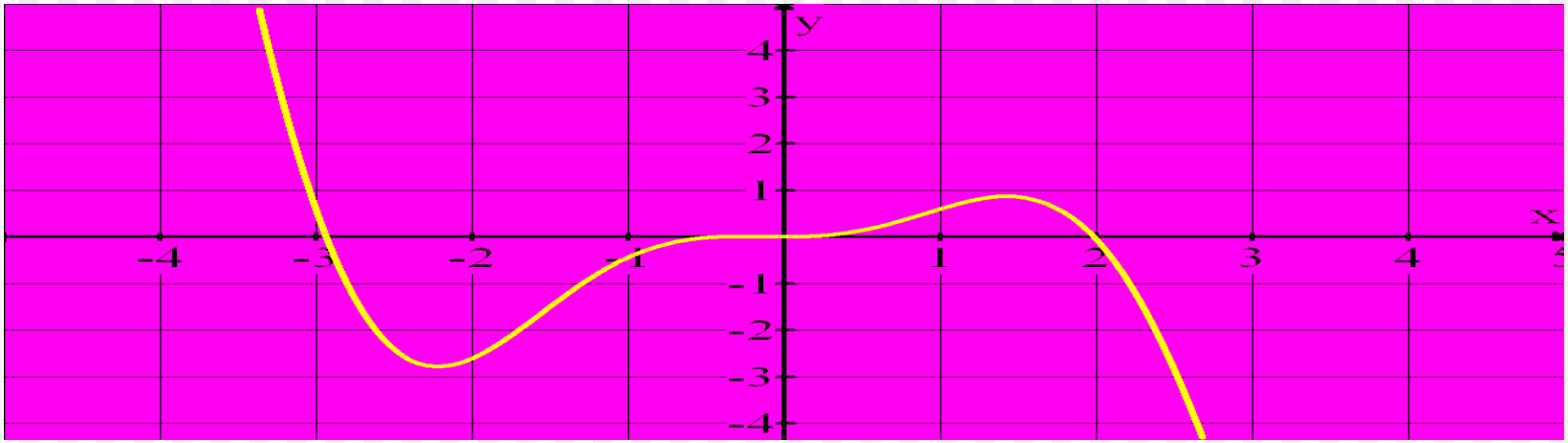
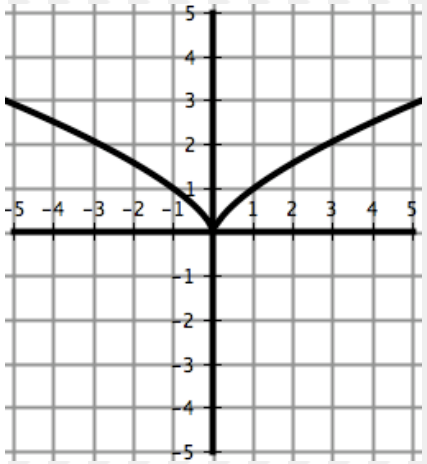




Continuity and Differentiability

- Graph the derivatives of the following function:



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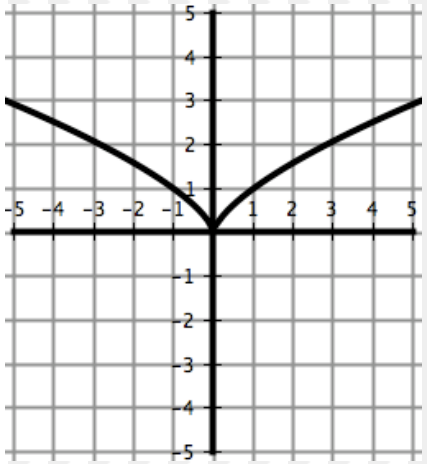
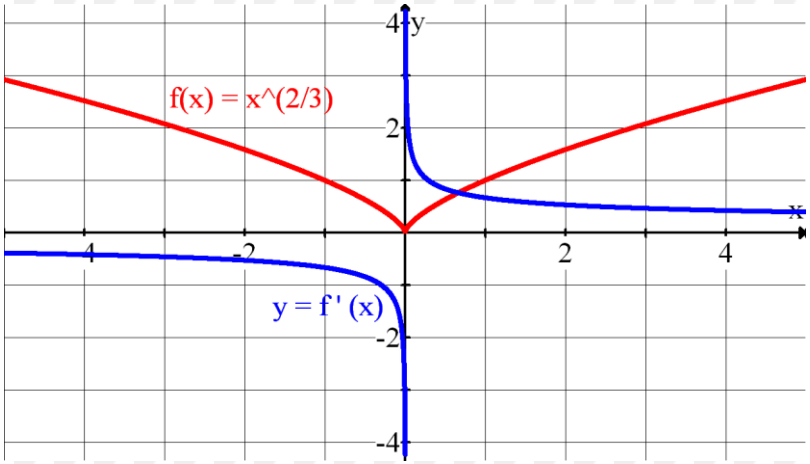
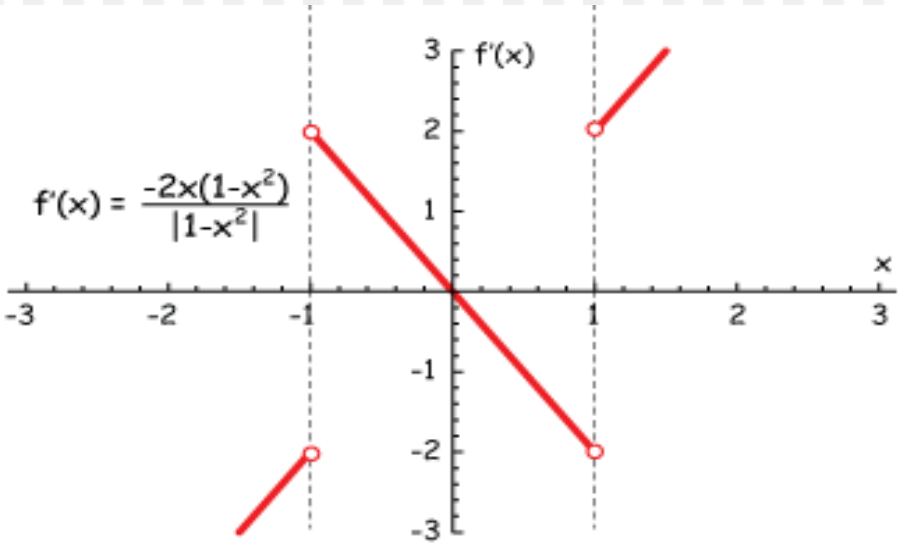




- Continuous functions are non-differentiable under the following conditions:
  - The function has a “corner”
  - The function has a “cusp”
  - The function has a vertical tangent
- This non-differentiability can be seen in that the graph of the derivative has a discontinuity in it!

Continuity and Differentiability

- Graph the function given its derivative:



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