Sections 4.6

Integration with Logarithmic and Exponential Functions

(and a few trig integrals!)

How do you solve $\int \frac{1}{x}\,dx$

Do you remember a function whose derivative is 1/x ? $\frac{d}{dx}[\ln x] = \frac{1}{x}$

Therefore $\boxed{\int \frac{1}{x}\,dx = \ln|x| + C}$

The domain of 1/x is $(-\infty, 0)$ $(0, \infty)$,
but the domain of ln x is $(0, \infty)$.
So we need to use absolute value.

Examples

$$\int_{-4}^{-2} \frac{1}{x}\,dx$$

$$\int \frac{x}{x^2+1}\,dx$$

$$\int \tan x \, dx$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

On Blue Sheets, memorize or not!

You try: $\int \cot x \, dx$

let $u = \sin x$

$du = \cos x \, dx$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$$

$$= \int \frac{du}{u} = ?$$

$$= \ln|u| + C = \ln|sinx| + C$$

$$\int \sec x \, dx$$

First, we multiply numerator and denominator by sec x + tan x:

$$\int \sec x \, dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$$

$u = \sec x + \tan x$,
$du = (\sec x \tan x + \sec^2 x)\, dx$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$\int (1/u)\, du = \ln |u| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

You try: $\int csc\, x\, dx$

$$\int \csc x \,\mathrm{d}x = \int \frac{\csc^2 x + \cot x \csc x}{\csc x + \cot x}\,\mathrm{d}x$$

$$u = \csc x + \cot x, \frac{\mathrm{d}u}{\mathrm{d}x} = -\csc^2 x - \cot x \csc x,$$

$$= -\int \frac{1}{u}\,\mathrm{d}u = -\ln|u|$$

$$= -\ln|\csc x + \cot x| + C$$

Using trig identities and log rules, you could see this rule in other forms. Such as

$$\ln\left|\csc x - \cot x\right| + C$$

Exponential Functions

$$\boxed{\int e^{x}\,dx = e^{x} + C}$$

$$\int e^{5x}\,dx$$

$$\boxed{\int a^x\,dx = \frac{a^x}{\ln a} + C}$$

$$\int_0^5 2^x\,dx$$

$$\int 3^{\sin\theta}\cos\theta\,d\theta$$

- Calculaugh 51/52

- P. 341 #41-51 odd,
- p. 351 #21, 23, 31-41odd, 51, 53, 57