

# AP Calculus AB and AP Calculus BC Curriculum Framework

The *AP Calculus AB and AP Calculus BC Curriculum Framework* specifies the curriculum — what students must know, be able to do, and understand — for both courses. AP Calculus AB is structured around three big ideas: limits, derivatives, and integrals and the Fundamental Theorem of Calculus. AP Calculus BC explores these ideas in additional contexts and also adds the big idea of series. In both courses, the concept of limits is foundational; the understanding of this fundamental tool leads to the development of more advanced tools and concepts that prepare students to grasp the Fundamental Theorem of Calculus, a central idea of AP Calculus.

## Overview

Based on the Understanding by Design (Wiggins and McTighe) model, this curriculum framework is intended to provide a clear and detailed description of the course requirements necessary for student success. It presents the development and organization of learning outcomes from general to specific, with focused statements about the content knowledge and understandings students will acquire throughout the course.

The **Mathematical Practices for AP Calculus (MPACs)**, which explicitly articulate the behaviors in which students need to engage in order to achieve conceptual understanding in the AP Calculus courses, are at the core of this curriculum framework. Each concept and topic addressed in the courses can be linked to one or more of the MPACs.

This framework also contains a **concept outline**, which presents the subject matter of the courses in a table format. Subject matter that is included only in the BC course is indicated with blue shading. The components of the concept outline are as follows:

- ▶ **Big ideas:** The courses are organized around big ideas, which correspond to foundational concepts of calculus: limits, derivatives, integrals and the Fundamental Theorem of Calculus, and (for AP Calculus BC) series.
- ▶ **Enduring understandings:** Within each big idea are enduring understandings. These are the long-term takeaways related to the big ideas that a student should have after exploring the content and skills. These understandings are expressed as generalizations that specify what a student will come to understand about the key concepts in each course. Enduring understandings are labeled to correspond with the appropriate big idea.
- ▶ **Learning objectives:** Linked to each enduring understanding are the corresponding learning objectives. The learning objectives convey what a student needs to be able to do in order to develop the enduring understandings. The learning objectives serve as targets of assessment for each course. Learning objectives are labeled to correspond with the appropriate big idea and enduring understanding.

- **Essential knowledge:** Essential knowledge statements describe the facts and basic concepts that a student should know and be able to recall in order to demonstrate mastery of each learning objective. Essential knowledge statements are labeled to correspond with the appropriate big idea, enduring understanding, and learning objective.

Further clarification regarding the content covered in AP Calculus is provided by **examples** and **exclusion statements**. Examples are provided to address potential inconsistencies among definitions given by various sources. Exclusion statements identify topics that may be covered in a first-year college calculus course but are not assessed on the AP Calculus AB or BC Exam. Although these topics are not assessed, the AP Calculus courses are designed to support teachers who wish to introduce these topics to students.

## Mathematical Practices for AP Calculus (MPACs)

The Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of AP Calculus students. They are drawn from the rich work in the National Council of Teachers of Mathematics (NCTM) Process Standards and the Association of American Colleges and Universities (AAC&U) Quantitative Literacy VALUE Rubric. Embedding these practices in the study of calculus enables students to establish mathematical lines of reasoning and use them to apply mathematical concepts and tools to solve problems. The Mathematical Practices for AP Calculus are not intended to be viewed as discrete items that can be checked off a list; rather, they are highly interrelated tools that should be utilized frequently and in diverse contexts.

The sample items included with this curriculum framework demonstrate various ways in which the learning objectives can be linked with the Mathematical Practices for AP Calculus.

The Mathematical Practices for AP Calculus are given below.

### MPAC 1: Reasoning with definitions and theorems

Students can:

- use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;
- confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
- apply definitions and theorems in the process of solving a problem;
- interpret quantifiers in definitions and theorems (e.g., “for all,” “there exists”);
- develop conjectures based on exploration with technology; and
- produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

**MPAC 2: Connecting concepts**

Students can:

- a. relate the concept of a limit to all aspects of calculus;
- b. use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
- c. connect concepts to their visual representations with and without technology; and
- d. identify a common underlying structure in problems involving different contextual situations.

**MPAC 3: Implementing algebraic/computational processes**

Students can:

- a. select appropriate mathematical strategies;
- b. sequence algebraic/computational procedures logically;
- c. complete algebraic/computational processes correctly;
- d. apply technology strategically to solve problems;
- e. attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- f. connect the results of algebraic/computational processes to the question asked.

**MPAC 4: Connecting multiple representations**

Students can:

- a. associate tables, graphs, and symbolic representations of functions;
- b. develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology;
- c. identify how mathematical characteristics of functions are related in different representations;
- d. extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
- e. construct one representational form from another (e.g., a table from a graph or a graph from given information); and
- f. consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

**MPAC 5: Building notational fluency**

Students can:

- a. know and use a variety of notations (e.g.,  $f'(x)$ ,  $y'$ ,  $\frac{dy}{dx}$ );

- b. connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);
- c. connect notation to different representations (graphical, numerical, analytical, and verbal); and
- d. assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

## **MPAC 6: Communicating**

Students can:

- a. clearly present methods, reasoning, justifications, and conclusions;
- b. use accurate and precise language and notation;
- c. explain the meaning of expressions, notation, and results in terms of a context (including units);
- d. explain the connections among concepts;
- e. critically interpret and accurately report information provided by technology; and
- f. analyze, evaluate, and compare the reasoning of others.