

47. (a) $\lim_{x \rightarrow 0^+} x^{1/x} = 0^\infty = 0$, not indeterminate

(See Exercise 105).

(b) Let $y = x^{1/x}$

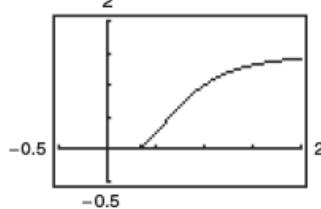
$$\ln y = \ln x^{1/x} = \frac{1}{x} \ln x.$$

Because $x \rightarrow 0^+$, $\frac{1}{x} \ln x \rightarrow (\infty)(-\infty) = -\infty$. So,

$$\ln y \rightarrow -\infty \Rightarrow y \rightarrow 0^+.$$

Therefore, $\lim_{x \rightarrow 0^+} x^{1/x} = 0$.

(c)



49. (a) $\lim_{x \rightarrow \infty} x^{1/x} = \infty^0$

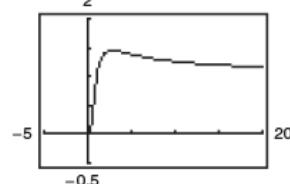
(b) Let $y = \lim_{x \rightarrow \infty} x^{1/x}$.

$$\ln y = \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0$$

So, $\ln y = 0 \Rightarrow y = e^0 = 1$. Therefore,

$$\lim_{x \rightarrow \infty} x^{1/x} = 1.$$

(c)



48. (a) $\lim_{x \rightarrow 0^+} (e^x + x)^{2/x} = 1^\infty$

(b) Let $y = \lim_{x \rightarrow 0^+} (e^x + x)^{2/x}$.

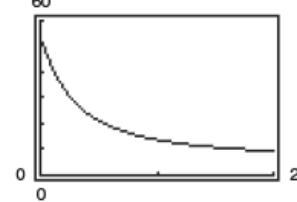
$$\ln y = \lim_{x \rightarrow 0^+} \frac{2 \ln(e^x + x)}{x}$$

$$= \lim_{x \rightarrow 0^+} \frac{2(e^x + 1)/(e^x + x)}{1} = 4$$

So, $\ln y = 4 \Rightarrow y = e^4 \approx 54.598$. Therefore,

$$\lim_{x \rightarrow 0^+} (e^x + x)^{2/x} = e^4.$$

(c)



50. (a) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = 1^\infty$

(b) Let $y = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$.

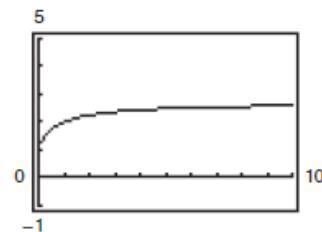
$$\ln y = \lim_{x \rightarrow \infty} \left[x \ln \left(1 + \frac{1}{x}\right) \right] = \lim_{x \rightarrow \infty} \frac{\ln[1 + (1/x)]}{1/x}$$

$$= \lim_{x \rightarrow \infty} \frac{\left[\frac{(-1/x^2)}{1 + (1/x)} \right]}{(-1/x^2)} = \lim_{x \rightarrow \infty} \frac{1}{1 + (1/x)} = 1$$

So, $\ln y = 1 \Rightarrow y = e^t = e$. Therefore,

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e.$$

(c)



51. (a) $\lim_{x \rightarrow 0^+} (1 + x)^{1/x} = 1^\infty$

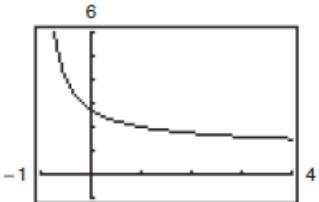
(b) Let $y = \lim_{x \rightarrow 0^+} (1 + x)^{1/x}$.

$$\begin{aligned}\ln y &= \lim_{x \rightarrow 0^+} \frac{\ln(1 + x)}{x} \\ &= \lim_{x \rightarrow 0^+} \left(\frac{1/(1+x)}{1} \right) = 1\end{aligned}$$

So, $\ln y = 1 \Rightarrow y = e^1 = e$.

Therefore, $\lim_{x \rightarrow 0^+} (1 + x)^{1/x} = e$

(c)



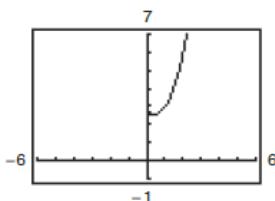
53. (a) $\lim_{x \rightarrow 0^+} [3(x)^{x/2}] = 0^0$

(b) Let $y = \lim_{x \rightarrow 0^+} 3(x)^{x/2}$.

$$\begin{aligned}\ln y &= \lim_{x \rightarrow 0^+} \left[\ln 3 + \frac{x}{2} \ln x \right] \\ &= \lim_{x \rightarrow 0^+} \left[\ln 3 + \frac{\ln x}{2/x} \right] \\ &= \lim_{x \rightarrow 0^+} \ln 3 + \lim_{x \rightarrow 0^+} \frac{1/x}{-2/x^2} \\ &= \lim_{x \rightarrow 0^+} \ln 3 - \lim_{x \rightarrow 0^+} \frac{x}{2} \\ &= \ln 3\end{aligned}$$

So, $\lim_{x \rightarrow 0^+} 3(x)^{x/2} = 3$.

(c)



52. (a) $\lim_{x \rightarrow \infty} (1 + x)^{1/x} = \infty^0$

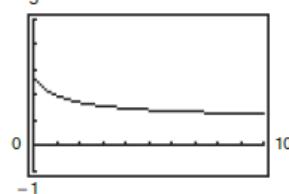
(b) Let $y = \lim_{x \rightarrow \infty} (1 + x)^{1/x}$.

$$\ln y = \lim_{x \rightarrow \infty} \frac{\ln(1 + x)}{x} = \lim_{x \rightarrow \infty} \left(\frac{1/(1+x)}{1} \right) = 0$$

So, $\ln y = 0 \Rightarrow y = e^0 = 1$.

Therefore, $\lim_{x \rightarrow \infty} (1 + x)^{1/x} = 1$.

(c)



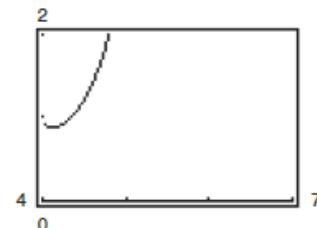
54. (a) $\lim_{x \rightarrow 4^+} [3(x - 4)]^{x-4} = 0^0$

(b) Let $y = \lim_{x \rightarrow 4^+} [3(x - 4)]^{x-4}$.

$$\begin{aligned}\ln y &= \lim_{x \rightarrow 4^+} (x - 4) \ln [3(x - 4)] \\ &= \lim_{x \rightarrow 4^+} \frac{\ln [3(x - 4)]}{1/(x - 4)} \\ &= \lim_{x \rightarrow 4^+} \frac{1/(x - 4)}{-1/(x - 4)^2} \\ &= \lim_{x \rightarrow 4^+} [-(x - 4)] = 0\end{aligned}$$

So, $\lim_{x \rightarrow 4^+} [3(x - 4)]^{x-4} = 1$.

(c)



55. (a) $\lim_{x \rightarrow 1^+} (\ln x)^{x-1} = 0^0$

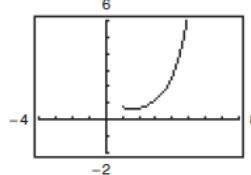
(b) Let $y = (\ln x)^{x-1}$.

$$\begin{aligned}\ln y &= \ln[(\ln x)^{x-1}] = (x-1)\ln(\ln x) \\ &= \frac{\ln(\ln x)}{(x-1)^{-1}}\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 1^+} \ln y &= \lim_{x \rightarrow 1^+} \frac{\ln(\ln x)}{(x-1)^{-1}} \\ &= \lim_{x \rightarrow 1^+} \frac{1/(x \ln x)}{-(x-1)^{-2}} \\ &= \lim_{x \rightarrow 1^+} \frac{-(x-1)^2}{x \ln x} \\ &= \lim_{x \rightarrow 1^+} \frac{-2(x-1)}{1 + \ln x} = 0\end{aligned}$$

Because $\lim_{x \rightarrow 1^+} \ln y = 0$, $\lim_{x \rightarrow 1^+} y = 1$.

(c)

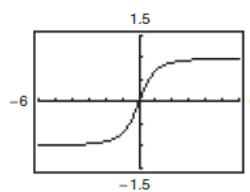


81. (a) Applying L'Hôpital's Rule twice results in the original limit, so L'Hôpital's Rule fails:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}} &= \lim_{x \rightarrow \infty} \frac{1}{x/\sqrt{x^2 + 1}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1}}{x} \\ &= \lim_{x \rightarrow \infty} \frac{x/\sqrt{x^2 + 1}}{1} \\ &= \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}\end{aligned}$$

$$\begin{aligned}(b) \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}} &= \lim_{x \rightarrow \infty} \frac{x/x}{\sqrt{x^2 + 1/x}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + 1/x^2}} = \frac{1}{\sqrt{1 + 0}} = 1\end{aligned}$$

(c)



56. $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$$(a) \lim_{x \rightarrow 0^+} \left[\cos\left(\frac{\pi}{2} - x\right) \right]^x = \lim_{x \rightarrow 0^+} [\sin x]^x = 0^0$$

(b) Let $y = (\sin x)^x$

$$\ln y = x \ln(\sin x) = \frac{\ln(\sin x)}{1/x}$$

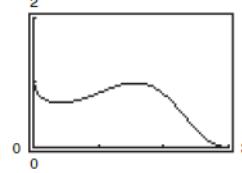
$$\lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{1/x} = \lim_{x \rightarrow 0^+} \frac{\cos x / \sin x}{-1/x^2}$$

$$= \lim_{x \rightarrow 0^+} \frac{-x^2 \cos x}{\sin x}$$

$$= \lim_{x \rightarrow 0^+} \frac{x}{\sin x} \left(\frac{-x \cos x}{1} \right) = 0$$

So, $\lim_{x \rightarrow 0^+} \left[\cos\left(\frac{\pi}{2} - x\right) \right]^x = 1$.

(c)



82. (a) Applying L'Hôpital's Rule twice results in the original limit, so L'Hôpital's Rule fails:

$$\lim_{x \rightarrow (\pi/2)^-} \frac{\tan x}{\sec x} \text{ is indeterminate: } \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow (\pi/2)^-} \frac{\tan x}{\sec x} = \lim_{x \rightarrow (\pi/2)^-} \frac{\sec^2 x}{\sec x \tan x}$$

$$= \lim_{x \rightarrow (\pi/2)^-} \frac{\sec x}{\tan x} \quad \left(\frac{\infty}{\infty} \right)$$

$$= \lim_{x \rightarrow (\pi/2)^-} \frac{\sec x \tan x}{\sec^2 x}$$

$$= \lim_{x \rightarrow (\pi/2)^-} \frac{\tan x}{\sec x}$$

$$\begin{aligned}(b) \lim_{x \rightarrow (\pi/2)^-} \frac{\tan x}{\sec x} &= \lim_{x \rightarrow (\pi/2)^-} \frac{\sin x}{\cos x} (\cos x) \\ &= \lim_{x \rightarrow (\pi/2)^-} \sin x = 1\end{aligned}$$

(c)

