Section 5.2 Exponential Growth and Decay, Newton's Law of Cooling

$y = y_0 e^{kt}$

1. A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 100 present at a given time and 300 present 5 hours later, how many will there be 10 hours after the initial given time? $(0, 100)$ $(5, 300)$

$300 = 100e^{5k}$ $\quad k = \frac{\ln 3}{5}$ $\quad y = 100e^{10k}$

$3 = e^{5k}$

$\ln 3 = 5k$ $\quad \boxed{y(10) = 900 \text{ bacteria}}$

2. In 1970 the population of a town was 2500, and in 1980 it was 3350. Assuming the population increases continuously at a constant rate proportional to the existing population, estimate the population in the year 2019. $(0, 2500)$ $(10, 3350)$

$3350 = 2500e^{10k}$ $\quad k = \frac{\ln 1.34}{10}$ $\quad y = 2500e^{49k}$

$1.34 = e^{10k}$

$\ln 1.34 = 10k$ $\quad \boxed{y(49) = 10{,}489.469 \text{ people}}$

3. If radioactive material decays continuously at aa rate proportional to the amount present, find the half-life of the material if after 1 year 99.57 percent of an initial amount still remains.

$.99.57y_0 = y_0 e^{1k}$ $\quad .50y_0 = y_0 e^{kt}$ $\quad t = \frac{\ln .5}{\ln .99\,57}$

$.99.57 = e^{k}$ $\quad .5 = e^{kt}$

$\ln .99\,57 = k$ $\quad \ln .5 = kt$ $\quad \boxed{t = 160.850 \text{ years}}$

Use Newton's Law of Cooling.

$T - T_s = (T_0 - T_s)e^{kt}$

4. Determine the reading on a thermometer 5 minutes after it is taken from a room at 72° (T_0) Fahrenheit to the outdoors where the temperature is 20° (T_s), if the reading dropped to 48° (T_1) after 1 minute.

$48 - 20 = (72 - 20)e^{1k}$ $\quad T - 20 = (72 - 20)e^{5k}$

$8 = 52e^{k}$ $\quad T = 20 + 52e^{5k}$

$8/52 = e^{k}$ $\quad k = \ln\left(\frac{8}{52}\right)$ $\quad \boxed{T(5) = 22.354°}$

5. An object in a room at 70° (T_s) cools from 350° (T_0) to 150° in 45 minutes. Find the time necessary for the object to cool to 80°.

$150 - 70 = (350 - 70)e^{45k}$ $\quad 80 - 70 = (350 - 70)e^{kt}$

$\boxed{t = 119.695 \text{ minutes}}$ $\quad 80 = 280e^{45k}$ $\quad 10 = 280e^{kt}$

$80/280 = e^{45k}$ $\quad k = \frac{\ln(8/28)}{45}$ $\quad \frac{1}{28} = e^{kt}$ $\quad \frac{\ln(1/28)}{k} = t$

6. Determine the outdoor temperature (T_s) if a thermometer is taken form a room where the temperature is 68° to the outdoors, where after ½ minute and 1 minute the thermometer reads 53° and 42°, respectively. next page →

7. When an object is removed from a furnace and placed in an environment with a constant temperature of 90° (T_s), its core temperature is 1500° (T_0). One hour after it is removed, the core temperature is 1120°. Find the core temperature 5 hours after the object is removed for the furnace.

$1120 - 90 = (1500 - 90)e^{1k}$ $\quad T_s - 90 = (1500 - 90)e^{5k}$

$1030 = 1410e^{k}$ $\quad T_s = 90 + 1410e^{5k}$

$\frac{1030}{1410} = e^{k}$ $\quad \boxed{= 383.298°}$

$\ln\left(\frac{103}{141}\right) = k$

6) $T_s = ?$

$T_0 = 68$

$T_{1/2} = 53$

$T_1 = 42$

$$53 - T_s = (68 - T_s)e^{\frac{1}{2}k} \qquad 42 - T_s = (68 - T_s)e^{k}$$

$$\frac{53 - T_s}{68 - T_s} = (e^{k})^{1/2} \qquad \frac{42 - T_s}{68 - T_s} = e^{k}$$

$$\frac{53 - T_s}{68 - T_s} = \left(\frac{42 - T_s}{68 - T_s}\right)^{1/2}$$

$$\left(\frac{53 - T_s}{68 - T_s}\right)^2 = \frac{42 - T_s}{68 - T_s}$$

$$(53 - T_s)^2 = (42 - T_s)\frac{(68 - T_s)^2}{68 - T_s}$$

$$(53 - T_s)^2 = (42 - T_s)(68 - T_s)$$

$$2856 - 110T + T_s^2 = 2809 - 106T_s + T_s^2$$

$$-4T_s = -47$$

$$\boxed{T_s = 11.75^\circ}$$

or

$$(42 - T_s) = (53 - T_s)e^{k30}$$

$$(53 - T_s) = (68 - T_s)e^{k30}$$

$$\frac{(42 - T_s)}{(53 - T_s)} = e^{k30} \qquad \frac{(53 - T_s)}{(68 - T_s)} = e^{k30}$$

$$\frac{(42 - T_s)}{(53 - T_s)} = \frac{(53 - T_s)}{(68 - T_s)}$$

$$(53 - T_s)^2 = (42 - T_s)(68 - T_s)$$

$$2856 - 110T_s + T_s^2 = 2809 - 106T_s + T_s^2 \qquad \boxed{T_s = 11.75^\circ}$$

$$-4T_s = -47$$