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59) $\frac{dy}{dx} = \frac{3\cos\theta\sin\theta + (2+3\sin\theta)\cos\theta}{3\cos\theta\cos\theta - (2+3\sin\theta)\sin\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{3(0)(1)+(5)0}{3(0)(0)-(5)(1)} = \frac{0}{-5} = 0$ Tangent line: $y=5$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = 0$ Tangent line: $y=1$

$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{3(-1)(0)+(2)(-1)}{3(-1)(-1)-2(0)} = \frac{-2}{3}$

$x = r\cos\theta$ $y = r\sin\theta$ Tangent Line:

$x = 2(-1)$ $y = 2(0)$ $y - 0 = -\frac{2}{3}(x+2)$

60) $r = 2 - 2\sin\theta$ $r' = -2\cos\theta$

$\frac{dy}{dx} = \frac{-2\cos\theta\sin\theta + (2-2\sin\theta)\cos\theta}{-2\cos\theta\cos\theta - (2-2\sin\theta)\sin\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{7\pi}{6}}$ is undefined Tan Line: $x = 3\cos 7\pi/6 \Rightarrow x = -1.5$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = 0$ Tan Line: $y = 4$

$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{-2(1)(0)+2(1)}{-2(1)(1)-2(0)} = \frac{2}{-2} = -1$ Tan Line:

$y - 0 = -1(x-2)$

$x = 2(1)$ $y = 2(0)$

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61) $r = 3 - 3\cos\theta \quad r' = 3\sin\theta$

$$\frac{dy}{dx} = \frac{3\sin\theta\sin\theta + (3-3\cos\theta)\cos\theta}{3\sin\theta\cos\theta - (3-3\cos\theta)\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \frac{3(1)(1) + (3-3\cdot 0)\,0}{3(1)(0) - (3-3\cdot 0)(1)} = \frac{3}{-3} = -1$$

$x = 3\cos\pi/2 \qquad y = 3\sin\pi/2$

$x = 0 \qquad y = 3$ eqn. tan. Line:

$y - 3 = -1(x - 0)$

62) $r = 3 - 2\cos\theta \quad r' = 2\sin\theta$

$$\frac{dy}{dx} = \frac{2\sin\theta\sin\theta + (3-2\cos\theta)\cos\theta}{2\sin\theta\cos\theta - (3-2\cos\theta)\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{2(0)(0) + (3-2(1))(1)}{2(0)(1) - (3-2\cdot 1)(0)} = \frac{1}{0} \text{ undefined}$$

$x = 1\cos(0) = 1$ eqn of tan line: $x = 1$

63) $r = 3\sin\theta \quad r' = 3\cos\theta$

$$\frac{dy}{dx} = \frac{3\cos\theta\sin\theta + 3\sin\theta\cos\theta}{3\cos\theta\cos\theta - 3\sin\theta\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/3} = \frac{3(\frac{1}{2})(\frac{\sqrt{3}}{2}) + 3(\frac{\sqrt{3}}{2})(\frac{1}{2})}{3(\frac{1}{2})(\frac{1}{2}) - 3(\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2})} = \frac{\frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}}{3/4 - 9/4} = \frac{6\sqrt{3}}{4}\cdot\frac{4}{-6}$$

$= -\sqrt{3}$

$x = 3(\sqrt{3}/2)(1/2) \qquad y = 3(\sqrt{3}/2)(\sqrt{3}/2)$

$x = 3\sqrt{3}/4 \qquad y = 9/4$

eqn. of tan line: $y - \frac{9}{4} = -\sqrt{3}\left(x - \frac{3\sqrt{3}}{4}\right)$

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(64) $r = 4 \quad r' = 0$

$$\frac{dy}{dx} = \frac{0\sin\theta + 4\cos\theta}{0\cos\theta - 4\sin\theta} = \frac{-\cos\theta}{\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

$x = r\cos\theta = 4(\sqrt{2}/2) = 2\sqrt{2}$

$y = r\sin\theta = 4(\sqrt{2}/2) = 2\sqrt{2}$

eqn of tan line: $y - 2\sqrt{2} = -1(x - 2\sqrt{2})$

(65) $r = 1 - \sin\theta \quad r' = -\cos\theta$

$$\frac{dy}{dx} = \frac{-\cos\theta\sin\theta + (1-\sin\theta)\cos\theta}{-\cos\theta\cos\theta - (1-\sin\theta)\sin\theta}$$

$c^2 = 1 - s^2$

Hor tan

$\frac{dy}{dx} = 0 \rightarrow dy/d\theta = 0$

$-\cos\theta\sin\theta + \cos\theta - \sin\theta\cos\theta = 0$

$\cos\theta(-2\sin\theta + 1) = 0$

$\cos\theta = 0 \quad -2\sin\theta + 1 = 0$

$\theta = \pi/2, 3\pi/2 \qquad 1/2 = \sin\theta$

$\theta = \pi/6, 5\pi/6$

$y = (1 - \sin\theta)\sin\theta$

$y = (1 - \sin 3\pi/2)\sin 3\pi/2$

$y = (1 - -1)(-1)$

$\boxed{y = -2}$

$y = (1 - 1/2)(1/2)$

$\boxed{y = 1/4}$

Cusp at $\theta = \pi/2$

Vert Tan

$\frac{dy}{dx}$ und. $\rightarrow \frac{dx}{d\theta} = 0$

$-\cos^2\theta - \sin\theta + \sin^2\theta = 0$

$-(1 - \sin^2\theta) - \sin\theta + \sin^2\theta = 0$

$-1 + \sin^2\theta - \sin\theta + \sin^2\theta = 0$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0 \qquad \sin\theta - 1 = 0$

$\sin\theta = -1/2 \qquad \sin\theta = 1$

$\theta = 7\pi/6, 11\pi/6 \qquad \theta = \pi/2$

$x = (1 - -1/2)\cos 7\pi/6$

$x = 3/2(-\sqrt{3}/2)$

$\boxed{x = -3\sqrt{3}/4}$

$x = 3/2(\cos 11\pi/6)$

$x = 3/2(\sqrt{3}/2)$

$\boxed{x = 3\sqrt{3}/4}$

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67) $r = 2\csc\theta + 3 \quad r' = -2\csc\theta\cot\theta$

$$\frac{dy}{dx} = \frac{-2\csc\theta\cot\theta\sin\theta + (2\csc\theta + 3)\cos\theta}{-2\csc\theta\cot\theta\cos\theta - (2\csc\theta + 3)\sin\theta}$$

Hor tan

$\frac{dy}{dx} = 0 \rightarrow \frac{dy}{d\theta} = 0 \qquad -2\cot\theta + 2\cot\theta + 3\cos\theta = 0$

$3\cos\theta = 0$

$\cos\theta = 0$

$\theta = \pi/2,\ 3\pi/2$

$\theta = \pi/2$

$x = (2\csc\theta + 3)\cos\theta$

$x = [2(1) + 3]\,0$

$x = 0$

$y = [2(+1) + 3](+1)$

$y = 5$

Hor. Tan at $(0,5)$ or $(5, \pi/2)$

eqn: $y = 5$

or

$\theta = \frac{3\pi}{2}$

$x = (2(-1) + 3)\,0$

$x = 0$

$y = (2(-1) + 3)(-1)$

$y = -1$

at $(0,-1)$ or $(1, 3\pi/2)$

eqn: $y = -1$