There are times when we need to describe motion (or a curve) that is *not a function*.

We can do this by writing equations for the x and y coordinates in terms of a third

variable (usually *time* or $θ$ ).

Each parametric equation gives one of the coordinates of position at a certain time.

 $x=f\left(t\right) y=g(t)$

Parametric Equations can be used to answer questions of where and when.

**Example:** Write a set of parametric equations for the line $y=\frac{1}{2}x+3$

One possible answer: Another answer:

A: B:

Graphs look the same, but parametric equations show location at a certain time.

When t = 4, Graph A is at ( ) and Graph B is at ( )

Both are traveling along the same line, just B is traveling faster.

**Example:** $x=\sqrt{t} , y=t+1$

Make a table of values and sketch the curve, indicating the direction of the curve. Then eliminate the parameter.

|  |  |  |
| --- | --- | --- |
| **t** | **X** | **y** |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

**Example:** Graph the plane curve represented by the parametric equations:

$$x=3+2\cos(t), y=-1+3\sin(t ); 0\leq t\leq 2π$$



|  |  |  |
| --- | --- | --- |
| **t** | **x** | **y** |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Now eliminate the parameter.

**Derivatives**

$\frac{dy}{dt}=$ $\frac{dx}{dt}=$

$$\frac{dy}{dx}=$$

Given $\left\{\begin{array}{c}x=4\sin(t)\\y=2\cos(t)\end{array}\right.$ Find dx/dt, dy/dt, and dy/dx at t = $π/4$

**Example:** Describe the movement and write an equation of the tangent line when t = 1



**Example:** Find all the points of vertical and horizontal tangency.



**Second Derivatives**

Example: $x=\sin(t)$ Find dy/dx

 $y=t^{2}+1$

To find the second derivative of a parametrized curve, we find the derivative of the first derivative:

$$\frac{d^{2}y}{dx^{2}}=\frac{d}{dx}\left(y^{'}\right)=$$

**Example:**



**Example:**

Given the parametric equations $\left\{\begin{array}{c}x=\sqrt{t}\\y=\frac{1}{4}t^{2}-1\end{array}\right.$ Find the slope and concavity at (2, 3)

**Arc Length of a Parametric Equation**

Find the length of

